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Ranking-Based Statistical Localization for Wireless Sensor Networks

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***Abstract*—This paper studies Ranking-Based localization method, which, as a special case of common Range-Based localization methods, statistically estimates node positions using the ranking-order constraints rather than the actual values of network measurements. Through extensive empirical evaluation, the advantages of Ranking-Based approach over traditional Range-Based approaches under extreme network conditions are verified. Ranking-Based approach turns out to be more robust to measurement insufficiency and more immune to measurement noise.**

I. Introduction

Wireless sensor networks (WSNs) have attracted a lot of research interests in recent years [1]. In the applications of WSNs, positions of sensor nodes are critical prerequisite information for successful operation. The most ideal and accurate methods of obtaining node positions would be GPS or manual recording, both of which are unfortunately too expensive to be widely implemented in WSNs. Therefore, most efforts in localization have been devoted to the work which exploits inter-node proximities to calculate node coordinates [2].

Although remarkable progresses have been achieved in the so-called **Range-Free** methods which purely employ network constraints [3], [4], [5], **Range-Based** methods utilizing metric range measurements are still dominant in the field of high accuracy localization.

All Range-Based methods, no matter the deterministic algorithms as MDS-MAP [6] and DWMDS [7], or probabilistic ones as the constraint-based approach in [8] and "case-delete" MLE [9], are based on the common heuristic algorithm that maps the topology estimate X to a set of pairwise distance measurements between sensor nodes.

In realistic environment, it is generally hard for the distance measurement to exactly match the true Euclidean value, owing to the randomness and unpredictability of notorious wireless channels [10]. Localization qualities of Range-Based methods, as a result, are closely related to the channel conditions; once the radio channel is contaminated with large measurement noise, localization accuracy would be drastically deteriorated.

Meanwhile, under some other extreme network conditions, such as low node density, irregular network topology, large obstacles, etc., direct distance measurements may not be sufficient to support localization. In this situation, some Range-Based methods implement data inference techniques such as **Shortest Path First** algorithm [6] to estimate those unmeasured pairwise distances. Such crude inference unluckily could provide satisfactory results only in uniform and isotropic networks where the estimates don't obviously depart from true Euclidean distances.

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Our work intends to improve localization performance under those atrocious network conditions by providing a robust **Ranking-Based** method, as a special case and improvement of Range-Based methods. With a predefined signal model, locations of sensor nodes could be statistically estimated merely based on ranking-order information of inter-node distances. For our method doesn't directly apply metric measurement values, it has remarkable advantage over traditional Range-Based methods confronting the measurement insufficiency problem, and is more immune to the fluctuation of measurement noise.

The remaining of this paper is organized as follows. After an overview of traditional Range-Based methods, the system measurement model is established in Section II. Then in Section III, our novel ranking-based method is proposed in details with two different approaches. An extensive simulation follows in Section IV, where our method and traditional methods are evaluated under various network topologies, the results are particularly compared and analyzed. Finally, Section V makes a conclusion and possible future extensions are discussed.

II. Background

A. Problem Statement

Any Range-Based localization method, whatever measuring technique it employs, aims to minimize the Stress function defined as

$$Stress(X, \widehat{D}) = \sum_i \sum_j f\big(\delta_{ij}(X), \hat{d}_{ij}\big), \tag{1}$$

where X is the position estimation to be optimized, and $\{\delta_{ij}(X)\}$ are the corresponding Euclidean distances; $\widehat{D} = \{\hat{d}_{ij}\}$ are the disparities related to distance measurements $D = \{d_{ij}\}$. During the optimization, $\widehat{D}$ could be viewed as the "judgments" or "standards" for $\{\delta_{ij}(X)\}$ to map with.

Conclusively, any Range-Based localization problem consists of 3 major steps.

1) Build up the disparity set $\widehat{D} = \{\hat{d}_{ij}\}$ as the "judgments".

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2) Define the Stress function f as the metric of "closeness".
3) Optimize topology estimate X, so as to make the estimated Euclidean distances $D_{euc} = \{\delta_{ij}(X)\}$ approach the "judgments" $\widehat{D}$ as close as possible, in the sense of minimizing previous Stress function (1).

In traditional Range-Based methods, $\hat{d}_{ij}$s are usually constructed straight from range measurements $D = \{d_{ij}\}$, with the unique mapping through some transformation g

$$\hat{d}_{ij} = g(d_{ij}), \text{ for any } i,\ j \tag{2}$$

For example, with RSS [11] or TOA [12] technique implemented, $\hat{d}_{ij}$ and d_{ij} are related as $\hat{d}_{ij} = d_{ij}$, where d_{ij} is the metric-value distance measurement; in hop-count based methods such as DV-Hop [13], the transformation is $\hat{d}_{ij} = b \times d_{ij}$, where d_{ij} is the hop count and b is the average distance per hop.

To be distinguished from the following Ranking-Based method, all those traditional methods whose calculation include actual values of range measurements are categorized as *metric* localization.

The novelty of our Ranking-Based method lies in the 1^{st} major step above. Unlike metric methods, Ranking-Based method just employs the *ranking-order* information of distance measurements to construct $\widehat{D}$. Instead of applying an explicit and fixed g, this method here virtually admit any form of transformation, as long as its outcome $\hat{d}_{ij}$s still comply with the ranking-orders exhibited in D as

$$\text{if } d_{ij} \leq d_{kl}, \text{ then } \hat{d}_{ij} \leq \hat{d}_{kl}, \text{ for any } i,\ j,\ k,\ l \tag{3}$$

Without metric values of range measurements directly included, the Ranking-Based method is accordingly categorized as *non-metric* localization.

From the view of geometry, instead of mapping $\{\hat{d}_{ij}\}$ to a fixed point in the geometrical space as the metric method does, our Ranking-Based method just confines $\{\hat{d}_{ij}\}$ into a small region bounded by those ranking-order constraints. In this region, the optimal $\widehat{D}$ is found, along with the optimal X, to minimize the Stress function (1). Apparently, the way of localization has changed in Ranking-Based method, from an unconstrained optimization problem over X to a joint optimization problem over both X and $\widehat{D}$, where X is unconstrained and $\widehat{D}$ is constrained.

B. System Model

Before going into details about our proposed approach, it is necessary to firstly build up a measurement model to help us decide how ranking-order information could be appropriately applied.

In this work, we assume an indoor environment with RSSI (Radio Signal Strength Indicator) technique used in range measurement. The most accurate way to model radio channel in such environment is the empirical measurement with calibration. However, as our approach is model-independent, which works well with arbitrary signal model, for simplicity and to avoid obscurity, we employ the classical logarithmical path loss model proposed by Rapport [10], with only line-of-sight communication allowed.

In this model, the transmitted signal decays logarithmically during its propagation, subject to shadowing effect noise. Any distance measurement d_{ij} is straightforwardly restricted to accord with a log-normal distribution around the corresponding true Euclidean distance δ_{ij} with a variance σ_d^2 determined by the shadowing effect, that is,

$$f\big(d_{ij} \mid \delta_{ij}(X)\big) = \frac{1}{\sqrt{2\pi}\sigma_d d_{ij}} e^{-\frac{(\log d_{ij} - \log \delta_{ij})^2}{2\sigma_d^2}} \tag{4}$$

In the following context, $\delta_{ij}(X)$ would sometimes be denoted as δ_{ij} for the convenience of expression.

III. Ranking-Based Statistical Localization

With only ranking-order information employed, here we propose **Ranking Maximum Likelihood Estimation (RMLE)**, which uses MLE to statistically infer positions of sensor nodes.

To comprehensively evaluate RMLE, we use Case-delete MLE (CMLE) method in [9], which is a representative of traditional Range-Based methods, as a benchmark to facilitate our analysis.

A. CMLE

If different pairwise distance measurements are assumed to be independent, CMLE optimizes the topology estimate X to maximize the following loglikelihood

$$\ln L = \sum_i \sum_j \omega_{ij} \ln f\big(d_{ij} \mid \delta_{ij}(X)\big), \tag{5}$$

where

$$\omega_{ij} = \begin{cases} 1, & n_i \text{ and } n_j \text{ can communicate} \quad (6a) \\ 0, & n_i \text{ and } n_j \text{ can't communicate} \quad (6b) \end{cases}$$

As its name tells, CMLE deletes all unmeasured pairwise distances from its calculation.

Substituting measurement model (4) into the loglikelihood (5), the task of CMLE equivalently turns into minimizing the following Stress function

$$Stress = \sum_i \sum_j \omega_{ij} \big[\log d_{ij} - \log \delta_{ij}(X)\big]^2, \tag{7}$$

which equals to a mapping between logarithmic distances in the "Least Square Error" sense.

B. Basic Idea of RMLE

The Stress function of RMLE remains similar to CMLE, because the same MLE technique is adopted in its optimization. The only difference is that d_{ij}s are replaced by the unfixed disparities $\{\hat{d}_{ij}\}$, which conform the ranking-order constraints as (3)

$$Stress = \sum_i \sum_j \omega_{ij} \big[\log \hat{d}_{ij} - \log \delta_{ij}(X)\big]^2, \tag{8}$$

For localization, individual ranking-order relation as (3) provides far less information than the direct distance measurement

d_{ij}. Therefore, if the same *case-delete* approach is employed as $Stress$ (8) shows, RMLE would not provide comparable result to CMLE in most situations.

Hence, in this paper we prefer to optimize the Stress over a complete disparity matrix $\widehat{D} = \{\hat{d}_{ij}\}$ as

$$Stress = \sum_i \sum_j \left[\log \hat{d}_{ij} - \log \delta_{ij}(X) \right]^2, \tag{9}$$

with disparities corresponding to the unmeasured pairwise distances also included.

In order to achieve a complete $\widehat{D}$, Shortest Path First algorithm is employed to fill up all missing entries in the initial incomplete distance measurement matrix D with their shortest path estimates. Then a complete comparison of all entries in D is performed to construct the set of ranking-order constraints.

For unmeasured distances, their shortest path estimates are generally not accurate enough to be directly utilized in metric localization methods as CMLE. But here with only ranking-order information extracted, it is reasonable to assume that the monotonicity in (3) still holds between the shortest path distances $\{d_{ij}\}$ and the true inter-node distances $\{\delta_{ij}\}$. In most networks, if $\delta_{ij} \le \delta_{kl}$, such inequality would still be valid with high probability for corresponding shortest path distances as $d_{ij} \le d_{kl}$.

C. The Algorithm of RMLE

To reduce complexity and expense of the joint optimization problem of (9) over X and $\widehat{D} = \{\hat{d}_{ij}\}$, RMLE takes an iterative approach to optimize $\widehat{D}$ and X, reducing the Stress function in a monotonic fashion. The algorithm consists of the following steps.

1) After collecting all available distance measurements between neighboring nodes, shortest path is computed between any node pair in the network, with a complete shortest path distance matrix D generated.
2) Some crude but effective localization method such as MDS-MAP is employed on D to get a rough topology estimate $X^{(0)}$ as the initial value for the following iteration.
3) Optimal disparities $\widehat{D}^{(0)}$ is found for $D_{euc}(X^{(0)}) = \{\delta_{ij}(X^{(0)})\}$ to minimize Stress function
$$S^{(0)} = Stress\big(\widehat{D}^{(0)}, D_{euc}(X^{(0)})\big) \tag{10}$$
4) Iteration begins.
5) In the t^{th} iteration, update position estimates $X^{(t)}$
$$X^{(t)} = \min_X Stress\big(\widehat{D}^{(t-1)}, D_{euc}(X)\big) \tag{11}$$
6) Update disparity matrix $\widehat{D}^{(t)}$ for fixed $D_{euc}(X^{(t)})$.
$$\widehat{D}^{(t)} = \min_{\widehat{D}} Stress\big(\widehat{D}, D_{euc}(X^{(t)})\big) \tag{12}$$
The Stress of this iteration is then
$$S^{(t)} = Stress\big(\widehat{D}^{(t)}, D_{euc}(X^{(t)})\big) \tag{13}$$
7) If $\mid S^{(t-1)} - S^{(t)} \mid > \epsilon$, increase t by 1, go back to step 5; otherwise, iteration stops.

The key steps in this algorithm are finding optimal update $X^{(t)}$ and $\widehat{D}^{(t)}$ in each iteration.

With a fixed disparity matrix $\widehat{D}^{(t-1)}$ as "judgment", the optimization of $X^{(t)}$ is similar to CMLE, using the Newton-Raphson method in the minimization of Stress function. The estimated Euclidean distances are $D_{euc}(X^{(t)}) = \{\delta_{ij}^{(t)}\}$.

The optimization of $\widehat{D}^{(t)}$ is a bit more complicated, for the optimal $\widehat{D}^{(t)}$ should comply with all ranking-order constraints exhibited in D. If the ranking-order of $\delta_{ij}^{(t)}$s is exactly the same as that of d_{ij}s, we can simply choose $\hat{d}_{ij}^{(t)} = \delta_{ij}^{(t)}$ to define the optimal update; otherwise, the optimal update of $\hat{d}_{ij}^{(t)}$s is found by **Monotone Regression** with Kruscal's **up-and-down-blocks (UDB)** algorithm [14].

UDB finds a set of disparities $\widehat{D}^{(t)}$ which completely comply with all ranking-order constraints defined in D, with a minimum violation from $D_{euc}(X^{(t)})$ under the Stress function (13) at the same time.

Before UDB starts, $\widehat{D}^{(t)}$ is initialized with $D_{euc}(X^{(t)})$. At this moment, $S^{(t)}$ is at its minimum, yet $\hat{d}_{ij}^{(t)}$s don't completely comply with the ranking-order constraints. UDB then carries out some adjustments on $\hat{d}_{ij}^{(t)}$s' values.

Once d_{ij}s have been ordered from the smallest to largest in an ascending sequence denoted as $P = \{P(1), ..., P(\frac{N\times(N-1)}{2})\}$, where N is the number of sensor nodes in the network, corresponding sequence Q could also be built up for $\hat{d}_{ij}^{(t)}$s in the same index order, where

$$P(k) = d_{ij} \Rightarrow Q(k) = \hat{d}_{ij}^{(t)} \tag{14}$$

The monotonicity relationship is then checked between d_{ij}s and $\hat{d}_{ij}^{(t)}$s along P and Q. At any position q, if the monotonicity relationship holds, that is,

$$P(q-1) = d_{kl} \le P(q) = d_{ij} \tag{15}$$

$$Q(q-1) = \hat{d}_{kl}^{(t)} \le Q(q) = \hat{d}_{ij}^{(t)} \tag{16}$$

Then the corresponding disparities remain unchanged.

Otherwise, if monotonicity relationship is broken

$$P(q-1) = d_{kl} \le P(q) = d_{ij} \tag{17}$$

yet

$$Q(q-1) = \hat{d}_{kl}^{(t)} > Q(q) = \hat{d}_{ij}^{(t)} \tag{18}$$

Then the corresponding disparities have to be adjusted, $\hat{d}_{ij}^{(t)}$ should be larger, and $\hat{d}_{kl}^{(t)}$ should be smaller.

If we just change the values of $\hat{d}_{ij}^{(t)}$ and $\hat{d}_{kl}^{(t)}$, it is possible that $\hat{d}_{kl}^{(t)}$ would be smaller than its precedents in Q after the adjustment. Therefore, to maintain the ranking-order of Q which has already been established before position $q-1$, some of $\hat{d}_{kl}^{(t)}$'s precedents should also be engaged in the adjustment.

Using Q^{old} and Q^{new} to denote the sequence before and after adjustment respectively, we get the best solution which

TABLE I
A TYPICAL WORK PROCESS OF UDP ALGORITHM

Rank of d_{ij}	Initial $\hat{d}_{ij}$	Checking Position q 2	3	4	5	Final $\hat{d}_{ij}$
1	2.9	2.9	2.9	2.9	2.9	2.9
2	4.5	4.5	*3.1*	3.1	3.1	3.1
3	2.1	2.1	*3.1*	3.1	3.1	3.1
4	9.2	9.2	9.2	9.2	*6.1*	*5.8*
5	4.1	4.1	4.1	4.1	*6.1*	*5.8*
6	5.1	5.1	5.1	5.1	5.1	*5.8*

minimizes the increase on $S^{(t)}$ under the log-normal measurement model as

$$\hat{d}_{ij}^{(t)} = Q^{new}(q) = ... = Q^{new}(q-k)$$
$$= \left(Q^{old}(q)Q^{old}(q-1)...Q^{old}(q-k)\right)^{\frac{1}{k+1}} \quad (19)$$

where k is the smallest value meeting the following inequality

$$Q^{new}(q-k) \geq Q^{old}(q-k-1) \quad (20)$$

All other disparities not included in the above adjustment are left unchanged.

In this way, when the monotonicity checking reaches both ends of P and Q, we would have an optimal update of disparity matrix $\widehat{D}^{(t)}$.

TABLE I exhibits a typical iteration of UDP in a 4-node network, disparities adjusted in each step are shown in italic characters.

D. Directional Ranking-Based Method

The above Ranking MLE is based on a complete comparison between any two pairwise distances to globally optimize $\widehat{D}$ in each iteration, the complexity of UDB there is $O(N^4)$. When the network size extends, for even a moderate number of nodes (e.g. 50), there's a large number of pairwise distances (1225), resulting to an expensive job on complete comparison and global adjustment.

To reduce the complexity of comparison and adjustment in the previous method which could be denoted as Complete Ranking MLE (CRMLE), some modification is proposed here, called Directional Ranking MLE (DRMLE). In DRMLE, only distance pairs with a common ending vertex are compared, and the corresponding ranking-orders are included as constraints in the optimization. For example, ranking-orders as $\{d_{ij} \leq d_{il}\}$ would still be kept in the constraint set, while others as $\{d_{ij} \leq d_{kl}, i \neq k\}$ are discarded.

A most distinct advantage of DRMLE is that the task of UDP, including the comparison on D and the adjustment on $\widehat{D}^{(t)}$, could be distributed to different nodes and locally completed. When optimal $X^{(t)}$ has been achieved and spread throughout the network, each node n_i compares the distances between itself and all other nodes, that is $\{d_{i1}, ..., d_{iN}\}$; the comparison results compose the directional ranking-order constraint set C_i. Then $\{\hat{d}_{i1}, ..., \hat{d}_{iN}\}$ are locally updated with C_i using UDP. In this way, the complexity of UDP could be reduced to $O(N^3)$, or even virtually $O(N^2)$ when all comparing and adjusting operations are performed on every node parallelly.

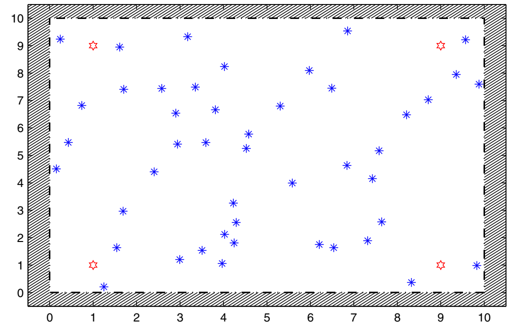

Fig. 1. Example of Square Topology with 50 nodes randomly deployed, red stars are anchors, blue asterisks are unknown nodes

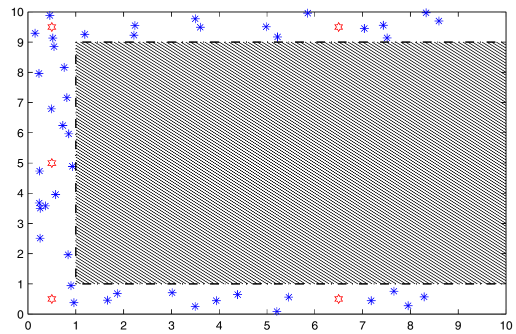

Fig. 2. Example of C-Shape Topology with 50 nodes randomly deployed, red stars are anchors, blue asterisks are unknown nodes

When implementing DRMLE in localization, different rows of $\widehat{D}$ are updated independently. It should be noted that the output $\widehat{D}$ is no longer strictly symmetric, $\hat{d}_{ij}$ could be unequal to $\hat{d}_{ji}$, because of the different adjusting strategies of row i and row j.

Although DRMLE waives a lot of available ranking-order information to reduce computation complexity, it has a close performance with CRMLE, as the following evaluation shows. This is a substantial evidence indicating that the directional ranking-order constraints contain more determinant information for node positions than other ranking-orders.

IV. SIMULATION RESULTS

The performances of CRMLE, DRMLE and CMLE are experimentally evaluated on different network topologies in Matlab R2007 with a 3.00GHz Pentium(R) 4 processor. Two typical topologies are considered: (a) 50 nodes randomly deployed in a $10m \times 10m$ square as Fig.1, representing the uncluttered environment; (b) 50 nodes randomly deployed in a C-Shaped corridor composed of 3 10-meter long, 1-meter wide branches as Fig.2, representing indoor networks with irregular and severely cluttered topology. In both figures, the shadows represent walls and obstacles. The signal transmission

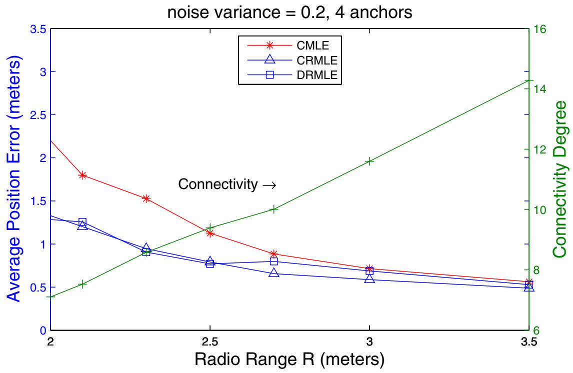

Fig. 3. Average position errors and connectivity against radio range R in square topology with 50 nodes randomly deployed, $\sigma_d^2 = 0.2$

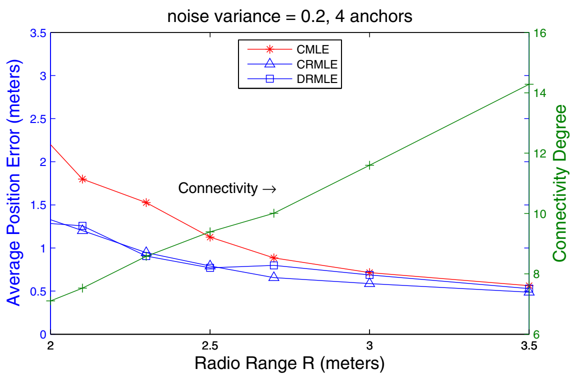

Fig. 4. Average position errors and connectivity against radio range R in C-Shape topology with 50 nodes randomly deployed, $\sigma_d^2 = 0.2$

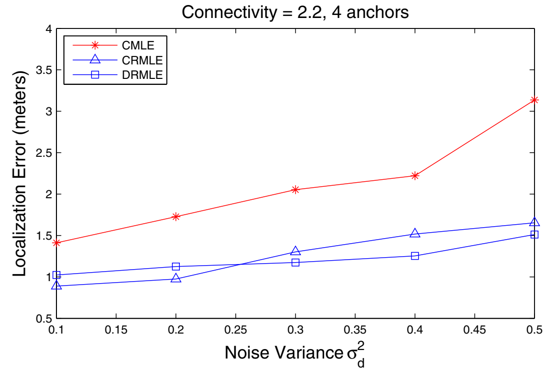

Fig. 5. Average position errors against σ_d^2 in square topology with 50 nodes randomly deployed, average connectivity is 5.7

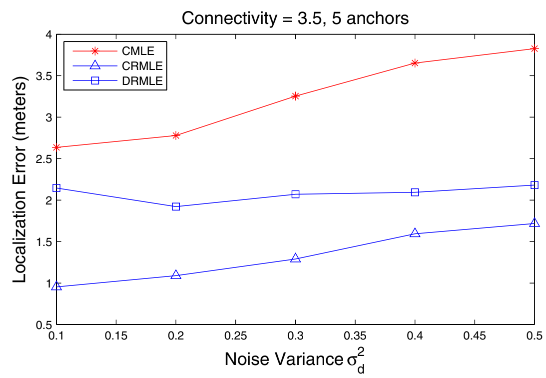

Fig. 6. Average position errors against σ_d^2 in C-Shape topology with 50 nodes randomly deployed, average connectivity is 8.9

is assumed to comply with the model established in Section II-B, subject to a log-normal noise.

A. The Influence of Network Connectivity

In the square topology and C-Shape corridor, Fig.3 and Fig.4 respectively exploit the influence of network connectivity on localization performances of the three methods, setting the variance of log-normal measurement noise σ_d^2 to 0.2. Each data point here represents the average result of 30 random trials.

In the square topology, the average network connectivity is completely controlled by sensor nodes' radio range R, and the average position errors of all three methods decrease monotonically with the growth of connectivity. Apparently, with more pairwise distances directly measured, the quality of localization will be remarkably improved, both in CMLE and RMLEs. When radio range $R > 3m$, or equivalently the average connectivity larger than 10, CMLE has comparable performance with both RMLE methods.

Nevertheless, when the network goes sparser as R decreases, the performance of CMLE is more severely influenced than those of RMLE methods, with its average position error rises at a faster pace. When $R = 2m$, the average position errors of CMLE comes to almost 2 times of RMLEs' results. Therefore, as RMLE methods have lower reliance on the distance measurements and are operated on a complete disparity matrix $\widehat{D}$, their performances are expected to be superior to CMLE in sparse networks with a low average connectivity.

In the cluttered C-Shape corridor, not only small radio range, but also the topology's irregularity renders significant difficulty for nodes to mutually communicate, especially those lying in different branches.

Although the performances of all three methods degrade simultaneously, the superiority of RMLE methods over CMLE is even more remarkable here. This is straightforward, as RMLE methods implement the complete disparity matrix $\widehat{D}$, they have taken the cross-branch inter-node connections into consideration; though the corresponding inferences on distances of these connections may not be accurate enough, they could still exploit some fundamental knowledge about the spatial relationship of nodes in different branches.

Moreover, in C-Shape topology, a notable gap appears between the performances of CRMLE and DRMLE. The directional ranking-order constraints turn out to be not abundant enough for localization here.

B. The Influence of Measurement Noise

Another advantage of RMLE over traditional metric CMLE is its immunity on the fluctuation of measurement noise.

In the square topology when the connectivity is 5.7, as Fig.5 shows, along with the increase of σ_d^2, the average position error of CMLE rises faster than those of RMLE methods. When σ_d^2 varies from 0.1 to 0.5, CMLE's average position error increases by 122%, compared to CRMLE's 86% and DRMLE's 48%, respectively.

The situation is similar in the C-Shape corridor with an average connectivity of 8.9, as shown in Fig.6.

Increasing measurement noise, the measurements of pairwise distances would severely depart from the true Euclidean values. However, as ranking-order constraints are functions of measured values, the influence of σ_d^2 on them could be alleviated to some extent. For example, the ranking-order of any two pairwise distances δ_{ij} and δ_{kl} would remain valid as long as the measurement error doesn't exceed $\mid \delta_{ij} - \delta_{kl} \mid$. Once the ranking-order stays correct, the information we implement in RMLE is accurate and effective like there's no measurement noise in the network.

With less ranking-order constraints employed, DRMLE provides a less accurate but more noise-insensitive localization result than CRMLE. Therefore, in networks where directional ranking-order constraints are sufficient, DRMLE would be a better choice, for its performance is more stable against measurement noise.

V. Conclusion

In this work, the efficacy of ranking-order constraints in the localization of WSNs is particularly analyzed.

Individually compared, a ranking-order constraint contains much less information than a metric distance measurement. As a result, such ranking-order constraint is commonly neglected in a lot of preceding methods where the measurement sufficiency is guaranteed, even though a lot of its advantages exist. For example, as it is usually easier to compare two distances than directly measure them, ranking-order constraints are much more facile to obtain. Meanwhile, for ranking-order constraint is virtually discrete variable but not consecutive value, it has good tolerance to measurement noise. When atrocious environment makes the direct distance measurements insufficient and unpromising, the implementation of ranking-order constraints would bring in significant gain on localization performance, as the evaluation exhibits.

As this is a pilot work in the field of Ranking-Based localization, a lot questions still remain open. For example, as we mentioned repeatedly above, although called *Ranking-Based*, our method is actually still a *Range-Based* method, for it employs the metric distance measurements in the calculation of shortest path distances and the initial topology estimate $X^{(0)}$. Future development of Ranking-Based methods could consider approaches which are completely measurement-independent, with ranking-order constraints obtained through other network observations such as the inter-node hop counts. Then the application scope of Ranking-Based methods could be further extended.

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