

## LETTER

# Achieving Fairness without Loss of Performance in Selection Cooperation of Wireless Networks\*

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**SUMMARY** This paper introduces a random selection cooperation scheme that takes the Decode-and-Forward (DF) approach to solve the unfairness problem in selection cooperation. Compared to previous work which obtained fairness but introduced performance loss, the proposed scheme guarantees fairness without performance loss. Its essence is to randomly select from the relays that can ensure the successful communication between the source and the destination, rather than to select the best relay. Both a theoretical analysis and simulation results confirm that the proposed scheme could achieve fairness and introduce no performance loss. We also discuss the conditions under which the proposed scheme is practical to implement.

**key words:** cooperative diversity, selection cooperation, decode-and-forward, outage probability, wireless networks

## 1. Introduction

In wireless networks, cooperative diversity [1], [2] is a distributed way to implement the multi-antenna technique that mitigates the fading effect over wireless channel. Without requiring sophisticated and expensive multiple antenna wireless nodes, it is affordable and enjoys higher flexibility.

In the scenario where multiple relays between the source and the destination are available, selection cooperation [3], [4] shows the merits created by its simplicity, i.e. it does not require precise synchronization among the distributed nodes nor exact Channel Side Information (CSI) at the transmitters, unlike some other schemes such like distributed space-time coding and cooperative beamforming.

Selection cooperation schemes usually consist of two stages (time slots). In the first stage, the source broadcasts the signal to all the relays (and may or may not include the destination, depending on the network scenario). In the second stage, a best relay [3], [4] is selected to retransmit to the destination. As a result, the relays may have different *average probabilities of being selected* due to their distributed nature. This will cause unfairness among the relays. For example, in the energy-constraint network, some nodes may

expend more power than the others. Some previous work [5], [6] tried to obtain fairness by selecting on the *weighted metric*. Although they can make the relays have equal probabilities of being selected, they introduced non-negligible performance loss compared to the *best selection* in [3], [4].

In this paper, we propose a random selection cooperation scheme that takes the Decode-and-Forward (DF) approach [1], [4] to solve the unfairness problem while maintaining the performance. The essence is to randomly select from the relays that can ensure the successful communication between the source and the destination, rather than to select the best relay. We use outage probability as the performance metric and show that the proposed scheme has the same outage probability as the best selection in [4]. And by adjusting the probabilities of the random selection, the relays can have equal probabilities of being selected. Simulation results confirm that the proposed scheme could maintain fairness without performance loss. We also discuss the conditions when for the proposed scheme to be practically applicable.

## 2. System Model

We consider a general half-duplex wireless scenario with multiple relays, similar to the model that was widely adopted in [1]–[6]. The network consists of  $N + 2$  nodes: one source  $s$ , one destination  $d$  and  $N$  relays (labeled as  $1, 2, \dots, N$ ), and  $s$  and  $d$  can not communicate directly.

We consider independent quasi-static flat fading channels between the nodes. The instantaneous signal-to-noise ratio  $\gamma_{i,j}$  of the channel from node  $i$  to node  $j$  is

$$\gamma_{i,j} = \left(\frac{P}{N_0}\right) \cdot |h_{i,j}|^2 \quad (1)$$

where  $P$  is the transmit power,  $N_0$  is the additive white Gaussian noise power spectral density at the receiver, and  $h_{i,j}$  is the fading coefficient from node  $i$  to node  $j$  and can be modeled as a random variable with zero mean. And no direct communication link between  $s$  and  $d$  implies  $\gamma_{s,d} = 0$ . We suppose the transmit power is constant for all relays and the source  $s$ , and define  $\Gamma \triangleq \frac{P}{N_0}$  as the average signal-to-noise ratio (SNR). All nodes communicate in the DF fashion [1]. Similar to [1], [2], [4], we assume that node  $j$  can correctly decode the signal from node  $i$  if  $\gamma_{i,j} \geq \gamma_{th}$ , where  $\gamma_{th}$  is a predefined non-negative threshold.

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### 3. Scheme Description

Both the best selection scheme in [4] and the proposed scheme consists of two stages (time slots). In the first stage,  $s$  broadcasts its signal to all relays, which try to decode the signal from  $s$ . There is no difference between the two schemes in the first stage. And in the second stage, one relay  $r \in \{1, \dots, N\}$  is selected to retransmit to the destination  $d$ . The outage probability  $P_{\text{out}}$  defined as the probability that  $d$  fails in decoding the signal of  $s$  via  $r$ , is adopted as the performance metric.

#### 3.1 Relay Selection in the Second Stage

The differences between the best selection and the proposed scheme are in the second stage. In this section, first we will discuss the differences of the two schemes, then we will show that the proposed scheme has the same outage probability as the best selection scheme.

Before proceeding the details of the relay selection in the second stage, we define the *decoding set*  $D$  and the *selectable set*  $T$  as

$$D \triangleq \{i \mid \gamma_{s,i} \geq \gamma_{\text{th}}, i \in \Omega_N\} \quad (2)$$

$$T \triangleq \{i \mid \gamma_{s,i} \geq \gamma_{\text{th}}, \gamma_{i,d} \geq \gamma_{\text{th}}, i \in \Omega_N\} \quad (3)$$

where  $\Omega_j \triangleq \{1, 2, \dots, j\}$  ( $1 \leq j \leq N$ ). Hence  $D$  is the set of the relays which have decoded correctly after the first stage, and  $T$  is the set of relays which can ensure successful communication from  $s$  to  $d$  if the relay is selected to retransmit in the second stage. We call the relay  $i$  ( $i \in \Omega_N$ ) in  $T$  selectable, and define the probability  $\Pr\{i \in T\} = t_i$ . Hence the probability of a particular set  $T$  when  $T \neq \emptyset$  is

$$\Pr\{T \mid T \neq \emptyset\} = \frac{\prod_{i \in T} t_i \prod_{i \notin T} (1-t_i)}{1 - \prod_{i=1}^N (1-t_i)} \quad (4)$$

$T$  can be obtained by  $d$  after the first stage as follows: only the relays that have decoded correctly, i.e.  $i \in D$ , report to the destination  $d$ , then  $d$  can determine  $T$  by checking whether their instantaneous SNR  $\gamma_{i,d} \geq \gamma_{\text{th}}$  or not.

In the second stage of the best selection scheme,  $r$  is selected as the relay with the best channel to  $d$  from  $D$ , hence

$$r = \arg \max_{i \in D} \gamma_{i,d} \quad (5)$$

And the outage condition is that  $D = \emptyset$  or  $\gamma_{r,d} < \gamma_{\text{th}}$ .

In contrast, the proposed scheme randomly selects  $r$  from the selectable set  $T$  in the second stage, if  $T \neq \emptyset$ . Otherwise the outage is declared and no relay is selected. Therefore, we have

$$r \in \Omega_N \Leftrightarrow r \in T \Leftrightarrow T \neq \emptyset \quad (6)$$

and then the outage condition is that no relay is selectable, i.e.  $T = \emptyset$ . Hence the outage probability  $\mathbf{P}_{\text{out}}$  is

$$P_{\text{out}} = \Pr\{T = \emptyset\} \quad (7)$$

It is easy to see that the outage condition of the proposed

scheme is equivalent to that of the best selection scheme. And as a result, the proposed scheme has the same outage probability as the best selection in [4].

In the proposed scheme, the probability to select the relay  $i$  in the second stage depends on  $T$  and is given by  $\Pr\{r = i \mid T\}$ . Then we have

$$\Pr\{r = i \mid T\} = \begin{cases} \Pr\{r = i \mid T, T \neq \emptyset\} \triangleq \mathbf{P}_i(T) \geq 0, & \text{if } T \neq \emptyset \\ 0, & \text{if } T = \emptyset \end{cases} \quad (8)$$

Since  $r \in T$ ,  $\mathbf{P}_i(T) = 0$  if  $i \notin T$ . Define  $\mathbf{P}_T \triangleq \{\mathbf{P}_1(T), \dots, \mathbf{P}_N(T)\}$  as the *relay selection probability*.

$r$  must be selected according to  $\mathbf{P}_T$  when  $T \neq \emptyset$ , and  $\mathbf{P}_T$  must be set to guarantee fairness. That is, the relay  $i$ 's ( $i \in \Omega_N$ ) *average probability of being selected* in the second stage  $\Pr\{r = i\}$  must satisfy  $\Pr\{r = i\} = \Pr\{r = j\}$  ( $i, j \in \Omega_N$ ). And we define

$$\mathbf{P}_i \triangleq \Pr\{r = i \mid T \neq \emptyset\} = \Pr\{r = i \mid r \in \Omega_N\} \quad (9)$$

It is easy to see  $\Pr\{r = i\} = \Pr\{r = j\} \Leftrightarrow \mathbf{P}_i = \mathbf{P}_j = \frac{1}{N}$ . We then consider to obtain  $\mathbf{P}_i = \mathbf{P}_j = \frac{1}{N}$  instead. Using the total probability law, it is easy to see that

$$\mathbf{P}_i = \sum_{T \neq \emptyset} \mathbf{P}_i(T) \cdot \Pr\{T \mid T \neq \emptyset\} \quad (10)$$

#### 3.2 Relay Selection Probability $\mathbf{P}_T$

$\mathbf{P}_T$  can be calculated from (10). When  $N = 2$ , the calculation is quite simple. Let  $v_i$  ( $i = 1, \dots, N$ ) indicates whether a relay  $i$  is selectable or not, i.e.  $v_i = 1 \Leftrightarrow i \in T$  and  $v_i = 0 \Leftrightarrow i \notin T$ , and define  $V = \{v_1, \dots, v_N\}$ . Then we can equivalently write  $\mathbf{P}_i(T)$  with  $N = 2$  as

$$\mathbf{P}_i(T) = \mathbf{P}_i^v(v_1, v_2), (i = 1, 2) \quad (11)$$

If only one relay is selectable, i.e.  $T = \{i\}$ ,  $i = 1$  or  $2$ , the fairness of relay selection gives

$$\mathbf{P}_1^v(1, 0) = 1, \mathbf{P}_2^v(1, 0) = 0, \mathbf{P}_1^v(0, 1) = 0, \mathbf{P}_2^v(0, 1) = 1 \quad (12)$$

If both two relays are selectable, i.e.  $T = \{1, 2\}$ , combining (4)(10)(12), we have

$$\mathbf{P}_1 = \frac{t_1 \cdot (1-t_2) + t_1 \cdot t_2 \cdot \mathbf{P}_1^v(1,1)}{1 - (1-t_1) \cdot (1-t_2)}, \mathbf{P}_2 = \frac{t_2 \cdot (1-t_1) + t_1 \cdot t_2 \cdot \mathbf{P}_2^v(1,1)}{1 - (1-t_1) \cdot (1-t_2)} \quad (13)$$

The fairness of relay selection requires

$$\mathbf{P}_1 = \Pr\{r = 1 \mid r \in \Omega_2\} = \mathbf{P}_2 = \Pr\{r = 2 \mid r \in \Omega_2\} = \frac{1}{2} \quad (14)$$

and hence

$$\mathbf{P}_1^v(1, 1) = \frac{1}{2} + \frac{1}{2} \cdot \left(\frac{1}{t_1} - \frac{1}{t_2}\right), \mathbf{P}_2^v(1, 1) = \frac{1}{2} - \frac{1}{2} \cdot \left(\frac{1}{t_1} - \frac{1}{t_2}\right) \quad (15)$$

Since  $\mathbf{P}_i^v(1, 1) \in [0, 1]$  ( $i = 1, 2$ ), then  $t_i$  must satisfy

$$-1 \leq \frac{1}{t_1} - \frac{1}{t_2} \leq 1 \quad (16)$$

The scheme is practically applicable only when (16) holds, which is referred to as the *applicable condition*. That the applicable condition in (16) is not met indicates that there is too much difference between the two relays, and we can not

find the appropriate  $\mathbf{P}_1(T)$  and  $\mathbf{P}_2(T)$  to achieve fairness. For example, when  $t_1 = 1$  and  $t_2 = 0.4$ , the applicable condition is  $\frac{1}{t_1} - \frac{1}{t_2} = -1.5$ . Hence it is impossible to achieve fairness, that is, it is impossible to achieve  $\mathbf{P}_1 = \mathbf{P}_2$ . The detailed explanations are as follows. We have

$$\begin{aligned} \mathbf{P}_1 &= \Pr\{r = 1 | r \in \Omega_2\} \\ &= \Pr\{r = 1 | T = \{1\}\} \cdot \Pr\{T = \{1\}\} \\ &\quad + \Pr\{r = 1 | T = \{1, 2\}\} \cdot \Pr\{T = \{1, 2\}\} \\ &= \Pr\{r = 1 | T = \{1\}\} \cdot 0.6 + \Pr\{r = 1 | T = \{1, 2\}\} \cdot 0.4 \\ &= 1 \cdot 0.6 + \Pr\{r = 1 | T = \{1, 2\}\} \cdot 0.4 \\ &\geq 0.6 \end{aligned} \quad (17)$$

As a result,  $\mathbf{P}_1 > 0.5 > \mathbf{P}_2$ .

$\mathbf{P}_T$  with  $N > 2$  can be obtained similarly. However, the calculation becomes extremely difficult as  $N$  increases. Alternatively, we consider an  $N$  steps procedure to select  $r$ . Within each step, one element will be randomly selected from a relay  $c(k)$  and a set of relays  $\Omega_{c(k)-1}$  similar to the case of  $N = 2$  in (12)(15), where  $c(k) \triangleq N - k + 2$ . From this  $N$  steps procedure, we will see how to calculate  $\mathbf{P}_T$ .

If  $\Omega_j \cap T \neq \emptyset$ , then we call  $\Omega_j$  is *selectable*. We can also define an indicator  $v_{\Omega_j}$  to represent whether  $\Omega_j$  is selectable, where  $v_{\Omega_j} = 1 \Leftrightarrow \Omega_j \cap T \neq \emptyset$  and  $v_{\Omega_j} = 0 \Leftrightarrow \Omega_j \cap T = \emptyset$ . Let  $t_{\Omega_j} \triangleq \Pr\{\Omega_j \cap T \neq \emptyset\} = 1 - \prod_{j=1}^{N-1} (1 - t_j)$ , and  $r \in \Omega_j$  represents  $\Omega_j$  is selected in this  $N$  steps procedure.

The details of the  $N$  steps procedure are as follows.

**Step 1** If  $T = \emptyset$ , outage occurs. Otherwise, go to step 2.

**Step  $k$**  ( $2 \leq k \leq N$ ) In this step, we already have  $r \in \Omega_{c(k)}$  ( $2 \leq c(k) \leq N$ ). We will randomly select from  $\Omega_{c(k)-1}$  and  $c(k)$ . Like (14), we need to maintain

$$\begin{aligned} \Pr\{r = c(k) | r \in \Omega_{c(k)}\} &= \frac{1}{c(k)-1} \cdot \Pr\{r \in \Omega_{c(k)-1} | r \in \Omega_{c(k)}\} \\ &= \frac{1}{c(k)} \end{aligned} \quad (18)$$

where  $\Pr\{r \in \Omega_{c(k)-1} | r \in \Omega_{c(k)}\}$  and  $\Pr\{r = c(k) | r \in \Omega_{c(k)}\}$  are the average probabilities of  $\Omega_{c(k)-1}$  or  $c(k)$  of being selected in this step. The reason of (18) will be shown later<sup>†</sup>.

Regarding  $\Omega_{c(k)-1}$  and  $c(k)$  as two candidates for random selection, similar to the case of  $N = 2$  case in (11), denote the probabilities of the selection in this step as

$$\mathbf{Q}_{\Omega_{c(k)-1}}^v(v_{\Omega_{c(k)-1}}, v_{c(k)}) = \Pr\{r \in \Omega_{c(k)-1} | T, r \in \Omega_{c(k)}\} \quad (19)$$

$$\mathbf{Q}_{c(k)}^v(v_{\Omega_{c(k)-1}}, v_{c(k)}) = \Pr\{r = c(k) | T, r \in \Omega_{c(k)}\} \quad (20)$$

Similar to (12)(15), we have

$$\mathbf{Q}_{\Omega_{c(k)-1}}^v(1, 0) = 1, \quad \mathbf{Q}_{c(k)}^v(1, 0) = 0 \quad (21)$$

$$\mathbf{Q}_{\Omega_{c(k)-1}}^v(0, 1) = 0, \quad \mathbf{Q}_{c(k)}^v(0, 1) = 1 \quad (22)$$

and

$$\mathbf{Q}_{\Omega_{c(k)-1}}^v(1, 1) = \frac{1}{c(k)} + \frac{1}{c(k)} \cdot \left( \frac{c(k)-1}{t_{\Omega_{c(k)-1}}} - \frac{1}{t_{c(k)}} \right) \quad (23)$$

$$\mathbf{Q}_{c(k)}^v(1, 1) = \frac{c(k)-1}{c(k)} - \frac{1}{c(k)} \cdot \left( \frac{c(k)-1}{t_{\Omega_{c(k)-1}}} - \frac{1}{t_{c(k)}} \right) \quad (24)$$

Like (16), the applicable condition of this step is

$$-1 \leq \frac{c(k)-1}{t_{\Omega_{c(k)-1}}} - \frac{1}{t_{c(k)}} \leq c(k) - 1 \quad (25)$$

If  $\Omega_{c(k)-1}$  is selected in this step, go to the next step, otherwise  $r = c(k)$  and the selection is done.

From (18),  $\mathbf{P}_N = \Pr\{r = N | r \in \Omega_N\} = \frac{1}{N}$ , and  $\mathbf{P}_i$  ( $1 \leq i \leq N-1$ ) can be rewritten as

$$\begin{aligned} \mathbf{P}_i &= \Pr\{r = i | r \in \Omega_N\} \\ &= \Pr\{r = i, r \in \Omega_{N-1} | r \in \Omega_N\} \\ &= \Pr\{r = i | r \in \Omega_{N-1}\} \cdot \Pr\{r \in \Omega_{N-1} | r \in \Omega_N\} \\ &= \Pr\{r = i | r \in \Omega_i\} \cdot \prod_{j=i}^{N-1} \Pr\{r \in \Omega_j | r \in \Omega_{j+1}\} \\ &= \frac{1}{i} \cdot \prod_{j=i}^{N-1} \frac{j}{j+1} \\ &= \frac{1}{N} \end{aligned} \quad (26)$$

Therefore, (18) is a sufficient condition to achieve the fairness of relay selection.

From the  $N$  steps selection, we now explain the calculation of  $\mathbf{P}_T$ . We can rewrite  $\mathbf{P}_i(T)$  ( $i \in \Omega_N$ ) as

$$\begin{aligned} \mathbf{P}_i(T) &= \Pr\{r = i | T, r \in \Omega_N\} \\ &= \Pr\{r = i | T, r \in \Omega_i\} \cdot \Pr\{r \in \Omega_i | T\} \\ &= \mathbf{Q}_i^v(v_{\Omega_{i-1}}, v_i) \cdot \Pr\{r \in \Omega_i | T, r \in \Omega_{i+1}\} \cdot \Pr\{r \in \Omega_{i+1} | T\} \\ &= \mathbf{Q}_i^v(v_{\Omega_{i-1}}, v_i) \cdot \mathbf{Q}_{\Omega_i}^v(v_{\Omega_i}, v_{i+1}) \cdot \Pr\{r \in \Omega_{i+1} | T\} \\ &= \mathbf{Q}_i^v(v_{\Omega_{i-1}}, v_i) \cdot \prod_{j=i}^{N-1} \mathbf{Q}_{\Omega_j}^v(v_{\Omega_j}, v_{j+1}) \end{aligned} \quad (27)$$

From (27), Algorithm 1 summarizes the calculation of  $\mathbf{P}_T$ .

<p><b>Input</b> : <math>V, \mathbf{Q}_{\Omega_n}^v(1, 1), \mathbf{Q}_{n+1}^v(1, 1)</math> (<math>1 \leq n \leq N-1</math>)</p> <p><b>Output</b>: <math>\mathbf{P}_T</math></p> <pre style="margin: 0;"> 1  if <math>\sum_{j=1}^N v_j = 0</math> then // Step 1 2      <math>\mathbf{P}_T \leftarrow \mathbf{0}</math>; // <math>T = \emptyset</math>, none is selectable 3      return ; 4  end 5  <math>\mathbf{P}_T \leftarrow \mathbf{1}</math>; // <math>T \neq \emptyset</math>, the calculation begins 6  for <math>i \leftarrow N</math> to 2 do // Step 2 to N 7      if <math>v_i = 0</math> then 8          // Only <math>\Omega_{i-1}</math> is selectable 9          <math>\mathbf{P}_i(T) \leftarrow 0</math>; 10     else if <math>\sum_{j=1}^{i-1} v_j = 0</math> then 11         // Only <math>i</math> is selectable 12         <math>\mathbf{P}_j(T) \leftarrow 0</math> (<math>\forall j \in \Omega_{i-1}</math>); 13         return ; 14     else 15         // Both are selectable 16         <math>\mathbf{P}_i(T) \leftarrow \mathbf{P}_i(T) \cdot \mathbf{Q}_i^v(1, 1)</math>; 17         <math>\mathbf{P}_j(T) \leftarrow \mathbf{P}_j(T) \cdot \mathbf{Q}_{\Omega_{i-1}}^v(1, 1)</math> (<math>\forall j \in \Omega_{i-1}</math>); 18     end 19 end</pre>
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**Algorithm 1:** The calculation of  $\mathbf{P}_T$ .

<sup>†</sup>Intuitively,  $\Omega_{c(k)-1}$  contains  $(c(k)-1)$  relays, hence it needs to have  $(c(k)-1)$  times the probability of a single relay  $c(k)$ .

Algorithm 1 can be executed at  $d$  after the first stage when  $T$  is obtained, then  $d$  can randomly select  $r$  according to  $\mathbf{P}_T$  in the second stage *without selecting in  $N$  steps*. Note that  $\mathbf{Q}_{\Omega_n}^v(1, 1)$  and  $\mathbf{Q}_{n+1}^v(1, 1)$  ( $1 \leq n \leq N-1$ ) can be calculated offline. This is because they only concern with  $t_i$  ( $i \in \Omega_N$ ), which only depend on the average channel condition.

### 3.3 The Discussion of the Applicable Condition

From (25), we know that all  $t_i$  ( $1 \leq i \leq N$ ) must satisfy the applicable conditions for the scheme to be practically applicable. But whether it is applicable depends not only on the values of  $t_i$ , but also on the arrangement of labels of the relays. For example, the network with  $t_1 = 0.4$ ,  $t_2 = 1$  and  $t_3 = 1$  does not meet the applicable condition. Since in step 2, the proposed scheme must select  $r$  from relay 1 and relay 2, but the applicable condition in this step is not met, which is same to the example in Sect. 3.2. But the network with  $t_1 = 1$ ,  $t_2 = 1$  and  $t_3 = 0.4$  does meet the applicable conditions in all steps, since it avoids selecting from two relays which have too much difference. As a result, analyzing the applicable condition is involved with the problem of the label arrangement, hence it is hard to obtain the necessary and sufficient applicable condition. However, we can still give out a very simple sufficient condition by the following Theorem 1. Though the sufficient condition in Theorem 1 is not necessary, it does not care about the arrangement of the labels, and it just needs to concern the  $t_i$  of each relay.

**Theorem 1.** *When  $t_i \geq 0.5$  ( $1 \leq i \leq N$ ), the applicable conditions of step  $k$  ( $2 \leq k \leq N$ ) can be met.*

*Proof.* When  $t_i \geq 0.5$  ( $1 \leq i \leq N$ ), we have  $1 - 0.5^{c(k)-1} \leq t_{\Omega_{c(k)-1}} \leq 1$  for  $2 \leq k \leq N$ . And since  $2 \leq c(k) \leq N$ , we have

$$c(k) - 3 \leq \frac{c(k)-1}{t_{\Omega_{c(k)-1}}} - \frac{1}{t_{c(k)}} \leq \frac{c(k)-1}{1-0.5^{c(k)-1}} - 1 \quad (28)$$

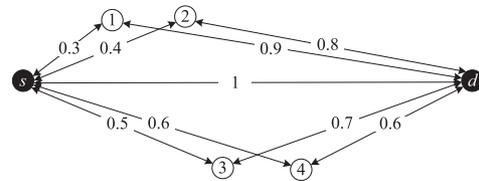
It is easy to get that  $c(k) - 3 \geq -1$  and  $\frac{c(k)-1}{1-0.5^{c(k)-1}} \leq c(k)$  when  $c(k) \geq 2$ . As a result, the applicable conditions in (25) of step  $k$  ( $2 \leq k \leq N$ ) can be met.  $\square$

When the channel condition is not too bad or the required bit rate is not too high, the sufficient condition in Theorem 1 can be met easily<sup>†</sup>.

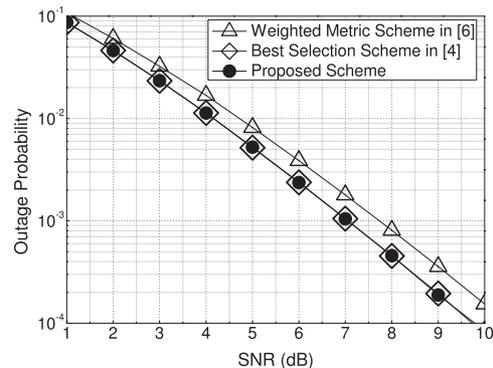
## 4. Simulation Results

In this section, we compare the proposed scheme with the best selection in [4] and the scheme on the weighted selection metric in [6] by simulation. Similar to [6], we consider the scenario in Fig. 1. The normalized distance between  $s$  and  $d$  is 1, and the distances from  $s$  to all the relays ( $d_{s,i}$ ) and from the relays to  $d$  ( $d_{i,d}$ ) are shown.

We assume all the nodes communicate over wireless channels with quasi-static and independent Rayleigh fading. Then  $h_{i,j}$  can be modeled as a zero-mean, independent, circularly symmetric complex Gaussian random variable with  $\mathbf{E}[|h_{i,j}|^2] = d_{i,j}^{-\alpha}$ , where  $\alpha$  is the path loss exponent and we



**Fig. 1** The simulation scenario.



**Fig. 2**  $P_{\text{out}}$  versus SNR ( $\Gamma$ ) when  $R = 0.75$  bit/s/Hz for three different selection cooperation schemes.

**Table 1** The times that the relays are selected in the three schemes when the simulation runs  $10^7$  times and  $\Gamma = 5$  dB. The Standard Deviation(SD) over mean of the times are also shown.

Relay	Proposed scheme	Scheme in [4]	Scheme in [6]
1	2484959	1013240	2471715
2	2485038	1735015	2504382
3	2491004	2852203	2506629
4	2486401	4399511	2517243
SD/Mean	$9.915 \times 10^{-4}$	$5.110 \times 10^{-1}$	$6.813 \times 10^{-3}$

suppose  $\alpha = 3$ . And we set  $\gamma_{\text{th}} = 2^{2-R} - 1$ , where  $R$  is the communication rate and 2 is the penalty of dividing the transmission into two stages. The probability  $t_i$  of each relay can be derived as

$$\begin{aligned} t_i &= \Pr\{i \in T\} = \Pr\{\gamma_{s,i} \geq \gamma_{\text{th}}, \gamma_{i,d} \geq \gamma_{\text{th}}\} \\ &= \Pr\{\gamma_{s,i} \geq \gamma_{\text{th}}\} \cdot \Pr\{\gamma_{i,d} \geq \gamma_{\text{th}}\} \\ &= \exp\left(-\gamma_{\text{th}} \cdot (d_{s,i}^\alpha + d_{i,d}^\alpha)\right) \end{aligned} \quad (29)$$

And we can get  $t_1 = 0.251003$ ,  $t_2 = 0.348829$ ,  $t_3 = 0.424984$  and  $t_4 = 0.453899$ .

Figure 2 shows the outage probability  $P_{\text{out}}$  versus the average signal-to-noise ratio  $\Gamma$  of the three schemes when  $R = 0.75$  bit/s/Hz. We can see that  $P_{\text{out}}$  of the proposed scheme is almost equal to that of the best selection in [4]. And both outperform the scheme in [6]. This confirms that the proposed scheme introduces no performance loss.

Table 1 shows the times that the relays are selected in

<sup>†</sup>We believe that analyzing the sufficient and necessary condition is very important and interesting. Since it is involved with the label arrangement problem and there are  $2^N$  sorts of arrangements with  $N$  relays, it will be very challenging. This may be the aim of our future work on the selection cooperation.

the three schemes when the simulation runs  $10^7$  times and  $\Gamma = 5$  dB. Similar to [5], we use the Standard Deviation (SD) over mean of the times as a rough metric of the fairness, since it can represent the unevenness of the times that the relays are selected. From Table 1, both the proposed scheme and the scheme in [6] make the relays have nearly equal times of being selected, and both their SD over mean of the times are negligible compared to that of the best selection in [4]. Thus, the fairness is achieved in both the proposed scheme and the scheme in [6].

## 5. Conclusion

In this paper, we introduced a random selection cooperation scheme with fairness. The essence is to randomly select from all the relays that can ensure the successful communication between the source and the destination in order to avoid the performance loss. By adjusting the probabilities of the random selection, the proposed scheme can make equal average probabilities of being selected of the relays. Both a theoretical analysis and simulation results

confirm that the proposed scheme can obtain fairness without performance loss.

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