

Approximation Algorithms for Link Scheduling with Physical Interference Model in Wireless Multi-hop Networks

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Abstract—The link scheduling in wireless multi-hop networks is addressed. Different from most of work that adopt the protocol interference model which merely take consideration of packet collisions, our proposed algorithms use the physical interference model to reflect the aggregated signal to interference and noise ratio (SINR), which is a more accurate abstraction of the real scenario. We first propose a centralized scheduling method based on the Integer Linear Programming (ILP) and resolve it by an approximate solution based on the randomized rounding method. The probability bound of getting a guaranteed approximate factor is given. We then extend the centralized algorithm to a distributed solution, which is favorable in wireless networks. It is proven that with the distributed scheduling method, all links can transmit without interference.

I. INTRODUCTION

In wireless multi-hop networks, concurrent transmissions that share a common channel may cause interference, and if too many devices transmit simultaneously, the interference will prevent an intended receiver from receiving the signal, and causes message loss. On the other hand, if too few nodes transmit at the same time, valuable bandwidth is wasted and the overall throughput may degenerate. Hence, the classic problem faced by the MAC layer scheduling protocols is to select an appropriate set of devices for concurrent transmissions, so that the interference does not cause message loss. In slotted wireless multi-hop networks, a natural and important goal of scheduling algorithms is to maximize the total throughput with the interference restriction.

Hence, an accurate model of interference is fundamental in order to derive theoretical or simulation-based results. There are two main interference models used, namely the protocol interference model and the physical interference models [1]. In the protocol model, a communication from node u to node v is successful if no other node within a certain interference range from v is simultaneously transmitting. Due to its simplicity and to the fact that it can be used to mimic the behavior of CSMA/CA networks such as IEEE 802.11, this model has been widely used in the literature. However, it doesn't reflect the advanced physical layer technologies that allow concurrent

multiple signal reception. In the physical interference model, a communication between nodes u and v is successful if the Signal to Interference and Noise Ratio (SINR) at receiver v is above a certain threshold. This model is less restrictive than the protocol interference model, and thus higher network capacity can be achieved by applying the physical interference model.

Clearly, the interference in the protocol model is a tremendous simplification of the physical reality faced in the wireless multi-hop networks. Particularly, the interference caused by different transmitters may accumulate and is not binary, i.e., does not vanish beyond any specific border. Moreover, a node may successfully receive a message even when there are other concurrently transmitting nodes in its interference range. In [2], [3], it is argued that the performance of protocol model based algorithms is inferior to those with more realistic and fundamental physical interference model. More recently, Moscibroda et al. [4] show experimentally that the theoretical limits of any protocol, which obeys the laws of protocol interference model, can be broken by a protocol explicitly defined for the physical interference model. However, although the physical interference model enjoy a much better performance, the scheduling problem under this model is notoriously hard to resolve.

In this paper, we study the link scheduling problem under the physical interference model. First, we formulate the problem as an Integer Linear Programming (ILP) problem, which is similar to the work in [5]. Since it is NP-complete [6], we get the approximate algorithm by using the randomized rounding method, and the probability bound of getting a guaranteed approximate ratio is given. Unlike the work in [5], we adjust the results of the rounding procedure based on all the constraints, and no iteration procedure is needed.

Since the centralized algorithm needs the global information of the network to do the scheduling, including the location of all nodes, the load of the links to be scheduled in a certain time slot, etc., collecting the required information may causes a lot of bandwidth overhead, especially when the network scale is large. Moreover, in some cases, the information is hard to get.

The difficulties in the implementation of centralized algorithm call for the distributed scheduling scheme that may achieve a fraction of the overall optimal performance. We then propose a distributed approach based on physical interference model. Each sending node implements the approach in three phases at each time slot, and all the constraints of the ILP problem in the centralized algorithm are satisfied.

The rest of the paper is organized as follows. After giving an overview of related work in Section II, the network and interference model is described in Section III. In Section IV, scheduling problem under physical interference model is formulated and we present the centralized algorithm to get the approximate solution. In section V, a distributed approach is proposed. Section VI presents the simulation results of the proposed algorithm and Section VII concludes the paper.

II. RELATED WORK

As mentioned in the previous section, an accurate modeling of interference is fundamental to the scheduling problem. Most of the scheduling mechanisms proposed for wireless multi-hop networks use the protocol interference model, such as [7]–[11]. These algorithms usually employ an implicit or explicit coloring strategy, and simply neglect the aggregated interference of nodes located farther away.

Only a few latest work have considered physical interference in the context [12]–[20], etc. In [12], Jain, et al. formulate the problem of scheduling under physical interference model as an LP problem. Unfortunately, no polynomial time solution and simulation-based evaluation of scheduling is given. [13] also provides an exponential-time LP formulation. In [14], Brar et al. present a heuristic scheduling method that is based on a greedy assignment of weighted links. Although it is based on physical interference model, the approximation factor of the algorithm is given only when nodes are uniformly distributed. The work of [15] considers physical interference in the minimum length scheduling problem. It uses a power-based interference graph model, which describes the interference relationship of every two links according to the SINR of the receiver. However, the model fails to consider the accumulation effect of interference. In [16], approximation algorithms for packet scheduling to minimize end-to-end delay with physical interference model are proposed. The works of [17], [18] study the problem of scheduling edges with SINR constraints to ensure that some property (e.g., connectivity) is satisfied. Similar with the work of [19], [20], they take power control, scheduling or routing together into account.

The work of [5] is similar to our centralized algorithm discussed in section IV. It also formulates the scheduling problem as ILP, but the randomized rounding procedure uses an iteration method which is more time-consuming than ours. Because the rounding procedure would not stop until a feasible solution is found, it costs a very long time to converge, and in some cases, it may not converge at all.

Various distributed algorithms have been proposed for finding good approximations of the scheduling problem based on the protocol interference model (e.g. [21]–[23], etc.). Only

a few previous work propose distributed algorithm based on the physical interference model. The work of [24]–[26] propose distributed algorithms based on the physical interference model, which is lattice-throughput-optimal. The work of [27] develops a constant-time distributed random access algorithm for scheduling and gives the performance bound of the algorithm. In [28], a distributed and randomized protocol is proposed, which uses physical carrier sensing to reduce message overhead. It is similar to our approach discussed in section V. But the protocol in [28] is sensitive to the scale of the network and spends much time to learn this information.

III. NETWORK AND INTERFERENCE MODELS

We abstract a wireless multi-hop network as a directed graph $G(V, E)$ where V is a set of vertices denoting the nodes comprising the network and E is a set of directed edges. The Euclidean distance between any two nodes $v_i, v_j \in V$, is denoted by $d(v_i, v_j)$. Let $e_{ij} \in E$ denotes the edge between v_i and v_j .

Each node is assumed to be equipped with a single transceiver working in the half-duplex way, and all nodes share a common channel. So a node can not send and receive packets simultaneously. All antennas are omnidirectional.

We use the physical interference model [1] to describe the interferences between active links. In this model, the successful reception of a transmission depends on the received signal strength, the interference caused by nodes transmitting simultaneously, and the ambient noise level. The received power $P_r(s_i)$ of a signal transmitted by sender s_i at an intended receiver r_i is

$$P_r(s_i) = P(s_i) \cdot g(s_i, r_i),$$

where $P(s_i)$ is the transmission power of s_i and $g(s_i, r_i)$ is the propagation attenuation (link gain). Given a sender-receiver pair (s_i, r_i) , r_i receives transmission successfully from s_i if and only if

$$SINR(r_i) = \frac{P(s_i) \cdot g(s_i, r_i)}{N + \sum_{s_j \in V \setminus \{s_i\}} P(s_j) \cdot g(s_j, r_i)}$$

where N is the ambient noise power level, and β is the minimum SINR threshold required for a successful message reception.

IV. THE CENTRALIZED APPROXIMATION ALGORITHM

In this section, we first formulate the link scheduling problem under the physical interference model as an Integer Linear Programming problem, then give an approximate algorithm using randomized rounding method, which can be done in polynomial time.

A. Problem formulation

We denote by $V^T \subseteq V$ and $V^R \subseteq V$ the set of transmitting and receiving nodes respectively. The time is divided into slots of fixed length and a frame is composed of T times slots. In time slot $t \in [1, T]$, there exists at least one edge being scheduled to transmit. We denote by $b_{ij}(e_{i,j} \in E)$ the total

traffic rate through link (i, j) and we use these to measure throughput. The goal of the scheduling method is to transmit all the edges in a frame to gain the maximum throughput.

To formally formulate the problem, the boolean variables x_{ij}^t is defined as

$$x_{ij}^t = \begin{cases} 1 & \text{if link } e_{ij} \text{ is scheduled to be transmit in slot } t \\ 0 & \text{otherwise} \end{cases}$$

Based on above assumptions, the linear integer programming formulation can be written as follows,

$$\max \frac{1}{T} \sum_{t=1}^T \sum_{e_{ij} \in E} b_{ij} x_{ij}^t$$

$$\text{s.t.} \quad \sum_{t=1}^T x_{ij}^t \geq 1, \quad \forall e_{ij} \in E \quad (1)$$

$$\sum_{v_i \in V^T} x_{ij}^t \leq 1, \quad \forall v_j \in V^R, \forall t \quad (2)$$

$$\sum_{v_j \in V^R} x_{ij}^t \leq 1, \quad \forall v_i \in V^T, \forall t \quad (3)$$

$$\sum_{v_j \in V^R} x_{ij}^t + \sum_{v_k \in V^T} x_{ki}^t \leq 1, \quad \forall v_i \in V^T \cap V^R, \forall t \quad (4)$$

$$\frac{P(v_i)g(v_i, v_j)x_{ij}^t + (1 - x_{ij}^t)\Delta}{N + \sum_{e_{kj} \in E \setminus \{e_{ij}\}} P(v_k)g(v_k, v_j)x_{kj}^t} \geq \beta, \quad \forall e_{ij} \in E, \forall t \quad (5)$$

$$x_{ij}^t \in \{0, 1\}, \quad \forall e_{ij} \in E, \forall t \quad (6)$$

The objective function of the formulation is to maximize the total network throughput, which is defined as the total traffic transmitted per slot. Constraint (1) guarantees that each active edge should be scheduled at least once. Constraint (2) and constraint (3) make sure that each node can only receive or send signal from or to one another node. This is because each node has only one transceiver. Under constraint (4), each node can not send and receive at the same time because it works in a half-duplex way. Constraint (5) expresses the required SINR threshold that should be satisfied in order to have a successful reception at the receiver. The item $(1 - x_{ij}^t)\Delta$ ensures that the inequality is also satisfied when link e_{ij} is not scheduled in time slot t (i.e. $x_{ij}^t = 0$), for a sufficiently large value of Δ .

B. Approximate algorithm with randomized rounding

Since ILP problems are NP-complete, there is no efficient algorithm known for solving them in bounded time (and there can not exist any unless $P = NP$). The above formulation does not help us solve the scheduling problem. However, it does guide us to a natural relaxation which helps to find a good approximate algorithm.

1) *Randomized rounding procedure*: We can use randomized rounding method to find efficient and near optimal solutions [29]. In this procedure, each variable x_{ij}^t will be relaxed to be in the range from zero to one and therefore converted from integer variables to fractional ones, i.e., $x_{ij}^t \in [0, 1]$, for $\forall e_{ij} \in E, \forall t$. The ILP problem is then relaxed to be a linear programming problem, which is called LP-relaxation. The optimal solution of the LP-relaxation can be treated as a probability vector with which we choose to include a link to a specific times lot. Denote the optimal solution as \hat{x}_{ij}^t . The rounding procedure can be described as follows,

$$x_{ij}^t = \begin{cases} 1, & \text{with probability } \hat{x}_{ij}^t \\ 0, & \text{with probability } 1 - \hat{x}_{ij}^t \end{cases} \quad (7)$$

However, after the rounding procedure, all the constraints may not be satisfied. To overcome this problem, the work in [5] uses a iteration method. The rounding procedure would not stop until a feasible solution is found, which costs a very long time to converge. Even worse, it may not find any feasible solution in the end. Unlike the work in [5], we adjust the results of the rounding procedure based on all the constraints, and no iteration procedure is needed. The detail of our proposed algorithm is discussed in the following.

2) *Approximate algorithm*: we denote by $\hat{\phi}^t = \{e_{ij} : x_{ij}^t = 1\}$ the set of edges that will be transmitted after the rounding procedure at time slot t . As the solution of the relaxed LP problem $\{\hat{x}_{ij}^t : \forall e_{ij} \in E, t \in T\}$ is the probability for the wireless links to transmit, we reorder the elements in $\hat{\phi}^t$ according to the probabilities in the non-decreasing order.

Algorithm 1 proceeds in two phases: phase 1 is corresponding to line 1 to 4; while phase 2 is corresponding to the rest of the algorithm. In phase 1, the ILP problem is relaxed to be a linear programming problem which is solved by the randomized rounding procedure. As mentioned before, there exists a problem that the rounding solution $\{x_{ij}^t : \forall e_{ij} \in E, t \in T\}$ may not satisfy all the constraints in the ILP problem. It can be resolved in phase 2. From line 5 to 17, it is checked slot by slot whether constraints (2) to (5) are satisfied. The solution of the relaxed LP $\{\hat{x}_{ij}^t : \forall e_{ij} \in E, t \in T\}$ can be viewed as the probability of the transmission of the links. The links with larger value of \hat{x}_{ij}^t have higher priority to transmit. So we check the links in a non-decreasing order of \hat{x}_{ij}^t . In each slot, if a link doesn't satisfy any of the constraints of (2),(3) or (4), it will not transmit in the current slot. After all links in $\hat{\phi}^t$ are checked, constraint (5) is checked from line 12 to 16. Because constraint (5) is the interference constraint, with which the transmission of a link is related to all the other transmitting links, it is checked after constraint (2) to (4) being checked. Constraint (1) guarantees that each active link should be scheduled at least once in a frame. However, after the procedure of randomized rounding and the check of constraint (2) to (4), x_{ij}^t of a link e_{ij} may be rounded to 0, or adjusted from 1 to 0, leading that constraint (1) is not satisfied. From line 18 to 25, constraint (1) is check. For a link e_{ij} that doesn't satisfy constraint (1), find the time slot t in which it has the largest probability to transmit (line 21). Then let

Algorithm 1 Centralized Approximate Algorithm

- 1: Consider all links in E ;
- 2: Relax the integer linear programming problem to be a linear programming problem (LP-relaxation);
- 3: Find the solutions of the relaxed LP $\{\hat{x}_{ij}^t : \forall e_{ij} \in E, t \in T\}$;
- 4: Do randomized rounding and get the rounding solution $\{x_{ij}^t : \forall e_{ij} \in E, t \in T\}$ from equation (7);
- 5: **for** each $t \in T$ **do**
- 6: Get the set $\hat{\phi}^t = \{e_{ij} : x_{ij}^t = 1\}$;
- 7: Consider all links $e_{ij} \in \hat{\phi}^t$ in non-decreasing order of \hat{x}_{ij}^t ;
- 8: **if** constraint 2 or constraint 3 or constraint 4 is not satisfied **then**
- 9: $x_{ij}^t = 0$;
- 10: Move e_{ij} away from $\hat{\phi}^t$;
- 11: **end if**
- 12: Consider all links $e_{ij} \in \hat{\phi}^t$ in non-decreasing order of \hat{x}_{ij}^t ;
- 13: **if** constraint 5 is not satisfied **then**
- 14: $x_{ij}^t = 0$;
- 15: Move e_{ij} away from $\hat{\phi}^t$;
- 16: **end if**
- 17: **end for**
- 18: Consider all links $e_{ij} \in E$;
- 19: **for** each link $e_{ij} \in E$ **do**
- 20: **if** $\sum_{t=1}^T x_{ij}^t < 1$ **then**
- 21: Consider the biggest element \hat{x}_{ij}^t for all $t \in T$;
- 22: $\hat{x}_{ij}^t = 1$;
- 23: Execute from line (5) to (17)
- 24: **end if**
- 25: **end for**

e_{ij} has probability 1 to transmit, which guarantees that e_{ij} is scheduled once. Constraint (1) is then satisfied.

3) *Performance analysis*: In this part, the performance of algorithm 1 is analyzed, including the complexity and the approximation ratio. It can be seen that algorithm 1 can finish in polynomial time, and the probability lower bound of being a $(1 - \theta)$ -approximation algorithm ($0 < \theta < 1$) is calculated.

The complexity of phase 1 depends on the algorithm solving the LP-relaxed problem. Let it be denoted by $O(P)$. Since it is a linear programming problem, it runs in polynomial time. Thus P is polynomial. Let N be the total number of the links in E . It can be easily seen that the complexity from line 5 to 17 in algorithm 1 is $O(NT)$, because there are two loops there. Similarly, the complexity from line 18 to 25 is $O(N^2T)$. Thus the complexity of the whole algorithm is $O(\max(P, N^2T))$, which is polynomial.

The approximate ratio of algorithm 1 can be got from the following theorem:

Theorem 1 (Approximate ratio of the centralized algorithm): For $\forall 0 < \theta < 1$ and $-\theta < \Delta A / \hat{A} < 1 - \theta$, the probability of algorithm 1 being $(1 - \theta)$ -approximate to the optimization

is lower bounded by $1 - e^{-\frac{(\theta + \frac{\Delta A}{\hat{A}})^2 \hat{A}}{2}}$, where \hat{A} is throughput calculated by the LP-relaxation, and ΔA is the variation of the throughput in phase 2.

Proof: In phase 1, let A_{rand} and \hat{A} be total throughput calculated by the randomized rounding in phase 1 and LP-relaxation respectively, i.e. $A_{rand} = \frac{1}{T} \sum_{t=1}^T \sum_{e_{ij} \in E} b_{ij} x_{ij}^t$, and $\hat{A} = \frac{1}{T} \sum_{t=1}^T \sum_{e_{ij} \in E} b_{ij} \hat{x}_{ij}^t$. From equation (7), we can get $E(x_{ij}^t) = \hat{x}_{ij}^t$. Thus,

$$\begin{aligned} E(A_{rand}) &= E\left(\frac{1}{T} \sum_{t=1}^T \sum_{e_{ij} \in E} b_{ij} x_{ij}^t\right) \\ &= \frac{1}{T} \sum_{t=1}^T \sum_{e_{ij} \in E} E(b_{ij} x_{ij}^t) \\ &= \hat{A} \end{aligned} \quad (8)$$

According to equation (8) and Chernoff Bound [29], we can get the following bound,

$$\begin{aligned} Pr(A_{rand} \geq (1 - \delta)\hat{A}) &= 1 - Pr(A_{rand} < (1 - \delta)\hat{A}) \\ &\geq 1 - e^{-\frac{\delta^2 \hat{A}}{2}}, \quad 0 < \delta < 1 \end{aligned} \quad (9)$$

Let A be the final solution of algorithm 1 and A_{Opt} be the optimal solution of the ILP problem. Obviously,

$$A \leq A_{Opt} \leq \hat{A} \quad (10)$$

In phase 2, the randomized rounding solution is checked whether it satisfies all the constraints in the ILP problem, and some adjustments are made, leading that the throughput changes. Suppose the throughput changes by ΔA after phase 2, and it follows that

$$A = A_{rand} + \Delta A \quad (11)$$

Substituting equation (11) into (9), we can derive

$$Pr(A \geq (1 - \delta + \frac{\Delta A}{\hat{A}})\hat{A}) \geq 1 - e^{-\frac{\delta^2 \hat{A}}{2}}, \quad 0 < \delta < 1 \quad (12)$$

In phase 2, the worst case is that all links are adjusted not to transmit, leading the throughput $A = 0$, i.e. $A_{rand} + \Delta A = 0$; while the best case is that after the adjustment, the solution approaches to the optimal, leading $A = A_{rand} + \Delta A \leq \hat{A}$. Thus we can get the variation range of ΔA as follows,

$$-A_{rand} \leq \Delta A \leq \hat{A} - A_{rand} \quad (13)$$

Let $\theta = \delta - \frac{\Delta A}{\hat{A}}$. Since $0 < \delta < 1$, it can be derived that $-\theta < \Delta A / \hat{A} < 1 - \theta$. Equation (12) can be written as follows,

$$Pr(A \geq (1 - \theta)\hat{A}) \geq 1 - e^{-\frac{(\theta + \frac{\Delta A}{\hat{A}})^2 \hat{A}}{2}}, \quad (14)$$

where $-1 + \frac{A_{rand}}{\hat{A}} \leq \theta \leq 1 + \frac{A_{rand}}{\hat{A}}$ (from equation (13)), and $-\theta < \Delta A / \hat{A} < 1 - \theta$.

Because $(0, 1) \subset [-1 + \frac{A_{rand}}{\hat{A}}, 1 + \frac{A_{rand}}{\hat{A}}]$, according to equation (10)(14), we can get

$$\begin{aligned} Pr(A \geq (1 - \theta)A_{Opt}) &\geq Pr(A \geq (1 - \theta)\hat{A}) \\ &\geq 1 - e^{-\frac{(\theta + \frac{\Delta A}{\hat{A}})^2 \hat{A}}{2}} \end{aligned} \quad (15)$$

where $\theta \in (0, 1)$, and $-\theta < \Delta A/\hat{A} < 1 - \theta$.

V. THE DISTRIBUTED ALGORITHM

The approximate algorithm discussed in section IV uses the physical interference model to represent the interference and has a good performance. However, as mentioned in section I, the centralized algorithm requires the global information of the network to do the scheduling. Collecting the required information may causes a lot of bandwidth overhead, especially when the network scale is large. Moreover, in some cases, the information is hard to get. The difficulties in the implementation of centralized algorithm call for the distributed scheduling scheme that may achieve a fraction of the overall optimal performance.

In this part, we extend the centralized algorithm 1 to a distributed solution, which takes the physical interference model into account and implements in three phases. It is assumed that nodes in the network can perform physical carrier sensing, and they can set the carrier sensing range to different values. As we show in the following part, by properly tuning the carrier sensing range, all the constraints of the ILP problem in section IV can be satisfied.

A. Carrier sensing range calculation

Let d_{max} and d_{min} be the maximum and minimum link length in the network respectively. The length diversity k is defined as

$$k = \lfloor \log(d_{max}/d_{min}) \rfloor \quad (16)$$

Suppose each node transmits at the same power P . Given a sender-receiver pair (s, r) , the link gain is modeled as $g(s, r) = d(s, r)^{-\alpha}$. The path-loss exponent α is a constant between 2 and 6. As the ambient noise power is much less than the interference power, it is omitted here. The following theorem describes how to tune the carrier sensing range for each node.

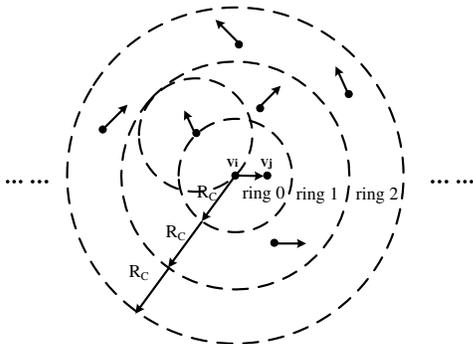


Fig. 1. The Euclidean plane is divided into a series of rings

Theorem 2 (Carrier sensing range): If the carrier sensing range is set to be $R_C = \rho 2^k$, the transmitting links in the network will not be interfered by all the other links, where ρ is defined as

$$\rho = 4(2\pi\beta \frac{\alpha - 1}{\alpha - 2})^{\frac{1}{\alpha}}, \quad (17)$$

α is the path-loss exponent and β is the minimum SINR threshold).

Proof: Without loss of generality, suppose link $e_{ij} \in E$ is transmitting data from node v_i to v_j . we normalize the minimum link length d_{min} to be 1. From the definition of the length diversity, we can get $2^k \leq d_{max} \leq 2^{k+1}$. The length of link e_{ij} satisfies $d_{ij} \leq d_{max} \leq 2^{k+1}$, thus the perceived power at v_j from v_i is at least

$$P_{ij} \geq \frac{P}{2^{(k+1)\alpha}} \quad (18)$$

The plane can be divided into a series of rings with the center located at sending node v_i , which can be seen in figure 1. They are denoted as ring 0, 1, \dots , respectively. Since the region of ring 0 is in the carrier sensing range of the sending node v_i , there are no other sending nodes locating in ring 0. For a sending nodes v_m located in ring 1, it must be at least $d_{mj} \geq R_C - 2^{k+1}$ away from v_j . Since each sending node must be outside the carrier sensing range of other sending nodes, there are at most $\pi / \arcsin(\frac{1}{2}) = 6$ sending nodes can transmit concurrently in ring 1. Consequently, the aggregated interference at v_j is

$$\sum_{l=1}^6 P_{lj} = \sum_{l=1}^6 \frac{P}{d_{lj}^\alpha} \leq \frac{6P}{(R_C - 2^{k+1})^\alpha}$$

In ring 2, the sending nodes are at least $2R_C - 2^{k+1}$ away from v_j . there are at most $\pi / \arcsin(\frac{1}{4}) \leq \frac{\pi}{1/4} = 4\pi$ sending nodes can transmit concurrently. So the aggregated interference at v_j is

$$\sum_{l=1}^{4\pi} P_{lj} \leq \frac{4\pi P}{(2R_C - 2^{k+1})^\alpha}$$

Similarly, in ring m , the sending nodes are at least $mR_C - 2^{k+1}$ away from v_j . there are at most $\pi / \arcsin(\frac{1}{2m}) \leq \frac{\pi}{1/2m} = 2m\pi$ sending nodes can transmit concurrently. So the aggregated interference at v_j is

$$\sum_{l=1}^{2m\pi} P_{lj} \leq \frac{2m\pi P}{(mR_C - 2^{k+1})^\alpha}$$

Consequently, the accumulated interference at the receiving node v_j is upper bounded by

$$\begin{aligned} I_{v_j} &\leq \sum_{m=1}^{\infty} \frac{2m\pi P}{(mR_C - 2^{k+1})^\alpha} \\ &= \frac{2\pi P}{2^{k\alpha}} \sum_{m=1}^{\infty} \frac{m}{(m\rho - 2)^\alpha} \\ &\leq \frac{2\pi P}{2^{k\alpha}} \sum_{m=1}^{\infty} \frac{m}{(m\rho/2)^\alpha} \end{aligned} \quad (19)$$

$$\leq \frac{2\pi P}{2^{(k-1)\alpha} \rho^\alpha} \frac{\alpha - 1}{\alpha - 2} \quad (20)$$

where (19) follows because $x - 2 > x/2, \forall x > 4$ and $\rho > 4$, given that $\beta \geq 1$ and $\alpha \geq 2$; and (20) follows from a bound

on Riemann's zeta function [6]. From equation (17), (18) and (20), we can get the $SINR$ at the receiving node v_j as follows:

$$SINR_{v_j} = \frac{P_{ij}}{I_{v_j}} \geq \frac{\frac{P}{2^{(k+1)\alpha}}}{\frac{2\pi P}{2^{(k-1)\alpha} \rho^\alpha} \frac{\alpha-1}{\alpha-2}} = \beta$$

B. Algorithm description

The frame structure is shown in Figure 2. A frame is composed of fixed length time slots. A time slot is divided into 3 phases, namely, carrier sensing phase, RTS-CTS phase and data-ack phase. Our distributed scheduling approach allows each node to implement it in each of the three phases. Next, we will present the details of the approach and show that after the three phases, all the constraints in the ILP problem are satisfied.

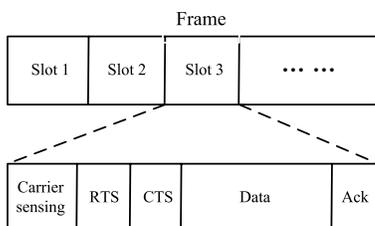


Fig. 2. Frame structure in distributed approach

1) *Carrier sensing phase*: The purpose of this phase is to guarantee that there are no other nodes sending data in the carrier sensing range of a transmitting node. Each node in the network sets its carrier sensing range to be R_C , which is calculated in theorem 2. So according to theorem 2, the node will transmit its data without any interference. Constraint (5) in the ILP is satisfied.

Consider a transmission slot starts at time t . The carrier sensing phase begins at the beginning of each time slot. It lasts for a period of t_c . Each node that wants to transmitting data randomly selects a time t_s and sends a *SENSING* signal, which lasts for a period of τ ($t \leq t_s \leq t + t_c - \tau$). The purpose of the *SENSING* signal is to let each node sense whether there are other transmitting nodes in its carrier sensing range. If a node does not sense any signal before sending the *SENSING* signal, it will occupy this time slot, and enter phase 2; otherwise, the node randomly selects another time slot and waits to transmit again. So the nodes located in each other's carrier sensing range will transmit in different slots, and no interference will occur. For each node, the purpose of randomly selecting a time to send *SENSING* signal is to prevent that all nodes send the *SENSING* signal concurrently, and they all will sense nothing.

If a link is scheduled, the sending node will not send the *SENSING* signal again until it can sense nothing, i.e., all the sending nodes that have been scheduled at least once will wait for the nodes that haven't been scheduled to transmit. This rule guarantees that all the links are scheduled at least once in a frame. So constraint (1) is satisfied.

2) *RTS-CTS phase*: In this phase, the sending node first sends RTS signal to the corresponding receiving node. If the receiving node receives RTS, it will respond CTS signal to the sending node. If the sending node receives CTS signal, it will enter phase 3. Otherwise, it will randomly select another time slot to transmit.

This phase guarantees that constraint (2) in the ILP is satisfied, i.e., a node can not receive data from more than one nodes simultaneously. An example can be seen in figure 3. Node A and B are both transmitting to node C. Since node B is outside the carrier sensing range of node A, A and B can transmit simultaneously in phase 1 without interference. Because C can not receive data from more than one node simultaneously (constraint 2), the data from A or B will not be received. Through the RTS-CTS procedure, A or B will transmit in another slot, and constraint (2) is satisfied.

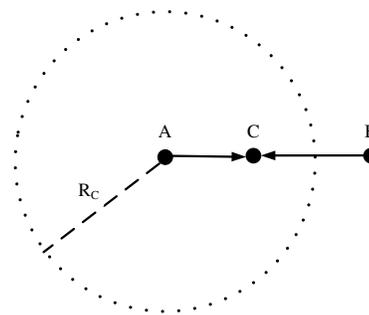


Fig. 3. An example in phase 2

3) *Data-ack phase*: In this phase, the sending node sends data to the receiving node, and waits for the ack signal from the receiving node. If the receiving node receives data correctly, it sends back ack signal to confirm the correct reception. If the sending node doesn't receive the ack signal, it will transmit the data again in the next slot.

Because of the distribution feature of our approach, a node can decide not to send data to more than one receiving nodes simultaneously. So constraint (3) is satisfied. Usually, the carrier sensing range is larger than the transmission range. Thus, when a node is transmitting, no other nodes will transmit to it. In addition, because of the half duplex character of wireless nodes, a node will not send data to others when it is receiving data. So constraint (4) is satisfied. In phase 1, constraint (1) and (5) are satisfied. Constraint (2) is satisfied in phase 2. So all the constraints are satisfied in our distributed approach, which can be viewed as a distributed solution of the ILP.

VI. SIMULATIONS

In this section, we present the simulation results that verify the performance of our proposed approximate algorithms in this paper.

A. Simulation scenario

We create a scenario where wireless nodes are uniformly distributed in a 100×100 square. In the simulation, we

normalize the transmission range of a node to be $R_T = 1$, and the interference range to be $R_I = 2.5$. For simplicity, all nodes are supposed to transmit at the same power level. We also assume $N = -90\text{dBm}$, $\beta = 10\text{dB}$, and $\alpha = 4$, which are similar with [2] and [24]. Let the frame length be $T = 100$. The whole simulation runs 100 times. In each simulation, n communication pairs are randomly selected. Suppose each of the selected links has the same traffic rate which is normalized to be 1.

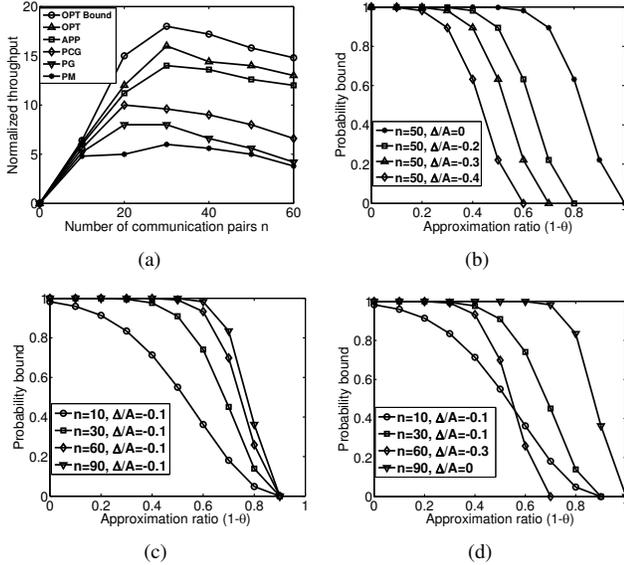


Fig. 4. The performance of the centralized algorithm

B. Centralized algorithm

We first compare the throughput performance between our proposed algorithm and several other centralized algorithms. The algorithm in [10] is based on protocol interference model and a polynomial-time coloring algorithm is used. To express conveniently, we call it Algorithm Protocol Model (PM). In [14], the algorithm is based on physical interference model, which is the same as ours. It is a heuristic scheduling method that is based on a greedy assignment of weighted links. We call it Algorithm Physical Greedy (PG). The authors in [15] construct the conflict graph using the physical interference model, which does not consider the accumulation effect either. It is called Algorithm Physical Conflict Graph (PCG). The throughput comparison of the several algorithms is shown in Fig.4(a). The notation OPT Bound here stands for the optimal solution of the LP-relaxation. The notation OPT is the optimal solution of ILP, which is got by traversal searching. Algorithm APP implies our approximate algorithm.

It can be seen that our proposed APP algorithm is well approximate to the optimal solution of ILP, which is a little lower than the upper bound. It also outperforms the protocol interference model based algorithm (PM algorithm) and other physical interference model based heuristic algorithms (PG and PCG algorithms). It performs ever better when more

nodes begin to transmit. The PM algorithm performs the worst, because it uses the protocol interference model, which is less precise than the physical interference model. When more and more nodes begin to transmit, the throughput drops for all the algorithms. This is because when interference is more severe, more transmissions begin to conflict with each other.

Then we evaluate the influence of the parameter $\Delta A/\hat{A}$ (defined in theorem 1) to the $(1-\theta)$ -approximate lower bound. Here we denote $\Delta A/\hat{A}$ by Δ/A in the legends of Fig. 4. It can be seen from Fig.4(b) that the lower bound decreases while $\Delta A/\hat{A}$ decreases, which is from the fact that more adjustments are made in the phase 2 of the algorithm 1. In our simulation, $\Delta A/\hat{A}$ is rarely smaller than -0.3, implying that the approximation ratio is larger than 0.5 with probability larger than 0.7, and the approximation ratio larger than 0.4 with probability larger than 0.9.

When more links begin to transmit, more adjustments are made in algorithm 1, i.e., ΔA decrease. The influence of the network scale n to the performance can be seen in Fig.4(c) and 4(d). We can get the following conclusions when n increases:

1) If $\Delta A/\hat{A}$ doesn't decrease, the probability lower bound increases. In Fig.4(c), $\Delta A/\hat{A}$ is fixed to be 0.1. The probability lower bound increases while more links begins to transmit. If $\Delta A/\hat{A}$ increases, the probability lower bound also increases (see plot ($n = 60, \Delta/A = -0.8$) and ($n = 90, \Delta/A = 0$) in Fig.4(c)).

2) If $\Delta A/\hat{A}$ decrease, the probability lower bound decreases. It can be seen in Fig.4(c)).

C. Distributed algorithm

In this simulation, nodes are not uniformly distributed, which is different from the scenario in the centralized algorithm. We select the transmission pairs properly to control the link length diversity k .

In Fig. 5, the throughput of the centralized algorithm and distributed approach are compared. It can be seen that the distributed algorithm approaches the centralized algorithm well when $k = 0$. The throughput of $k = 0$ and that of $k = 1$ are very close to each other; while it decreases sharply when $k = 2$. However, in real scenario k is not very large, typically $k \leq 2$. The distributed algorithm proposed in the paper is preferred to be implemented in the scenario that the link length diversity is small. The ideal scenario is that the nodes are located uniformly.

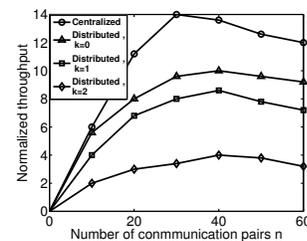


Fig. 5. The performance of distributed algorithm

VII. CONCLUSION

In this paper we study the link scheduling problem with the physical interference model, which is more accurate than the protocol interference model but notoriously hard to handle with. The scheduling problem is formulated to be a ILP problem, and we propose our centralized algorithm based on randomized rounding. We analyze the performance of the algorithm and give the probability bound of getting the guaranteed approximation ratio. As a distributed solution of the ILP problem, we also propose a distributed approach, which uses physical carrier sensing and implement in phases. Both algorithms performs well in the simulation.

ACKNOWLEDGMENT

This research is supported in part by the National Science Foundation of China (Grant No.60672107, 60672142 and 60772053), the Hi-tech Research and Development Program of China (Grant No.2006AA10Z261, 2006AA10A301, 2007AA100408 and 2009AA11Z103), the China 973 Project (Grant No.2007CB307100 / 2007CB307105).

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