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LETTER

CP-TDMA: Coloring- and Probability-Based TDMA Scheduling for Wireless Ad Hoc Networks

Xuedan ZHANG[†a)], *Member*, **Jun HONG**[†], **Lin ZHANG**[†], **Xiuming SHAN**[†], *and* **Victor O. K. LI**[††], *Nonmembers*

SUMMARY This paper addresses the issue of transmission scheduling in wireless ad hoc networks. We propose a Time Division Multiple Access (TDMA) scheduling scheme based on edge coloring and probabilistic assignment, called CP-TDMA. We categorize the conflicts suffered by wireless links into two types: explicit conflicts and implicit conflicts, and utilize two different strategies to deal with them. Explicit conflicts are avoided completely by a simple distributed edge-coloring algorithm μ-**M**, and implicit conflicts are minimized by applying probabilistic time slot assignments to links. We evaluate CP-TDMA analytically and numerically, and find that CP-TDMA, which requires only local information exhibits a better performance than previous work.

key words: *wireless ad hoc networks, TDMA scheduling, explicit conflict, implicit conflict, edge-coloring, probability theory*

1. Introduction

In this paper, we propose a novel distributed Time Division Multiple Access (TDMA) scheduling method based on the concepts of edge coloring and probabilistic assignment. We categorize the conflicts suffered by wireless links into two types: explicit conflicts and implicit conflicts, and design different strategies, derived from edge coloring and probabilistic assignment, respectively, to deal with them. To our best knowledge, this is the first paper which utilizes both edge coloring and probabilistic assignment in the domain of TDMA scheduling in wireless ad hoc networks. Our method, which requires only local information, gives a better performance than previous work.

The rest of the paper is organized as follows: Section 2 describes the network model and formally defines the problems. Section 3 presents the details of our scheduling algorithm. Section 4 evaluates the performance of CP-TDMA through computer simulations. Section 5 concludes the paper.

2. Model and Definitions

We represent a wireless ad hoc network as a directed graph $G = (V, E)$, where V is a set of vertices denoting the nodes comprising the network and E is a set of directed edges between vertices representing inter-node wireless links. Let $N = |V|$. Henceforth, the terms vertex and wireless node, as well as edge and wireless link, are used interchangeably in this paper.

We use the *protocol* interference model [1] to define the conditions for a successful wireless transmission, i.e., the transmission of a link between two nodes is successful if 1) the two nodes are within communication range of each other, 2) neither of the transceivers is being occupied by other links, and 3) no node within a receiving node's interference range is transmitting using the same time slot. We denote by Δ the maximum degree of a node in G and N and Δ are used as design parameters in our algorithm.

There are two kinds of conflicts suffered by wireless links in wireless ad hoc networks [2]. The first kind of conflicts, called *explicit conflicts*, is caused when more than one link share a common node's transceiver at the same time. The second kind of conflicts, called *implicit conflicts*, is a consequence of wireless interference in a neighborhood.

Consider the wireless network with a chain topology in Fig. 1. There are four nodes in the network and the distance between adjacent nodes is $100m$. The transmission range and the interference range of the nodes are $100m$ and $250m$ respectively. It is clear that the conflicts between $L1$ and $L2$ or between $L3$ and $L4$ are explicit conflicts and the conflicts between $L1$ and $L3$ or between $L2$ and $L4$ are implicit conflicts.

We use a distributed edge coloring algorithm which only requires local information, i.e. information from immediate neighbors, to assign different sub-frames to links with explicit conflicts. Avoiding implicit conflicts needs topology information from nodes out of transmission range and it may be difficult to get such information in wireless ad hoc networks. We use probability theory to analyze how to minimize the implicit conflicts without requiring any topology information. We shall elaborate on the details of our algorithm in the next section.

Fig. 1 A chain-topology wireless ad hoc network.

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[†] The authors are with the Department of Electronic Engineering, Tsinghua University, Beijing 100084, P.R. China.

[††] The author is with the Department of Electrical and Electronic Engineering, University of Hong Kong, Pokfulam, Hong Kong, and with the Department of Electronic Engineering, Tsinghua University, Beijing 100084, P.R. China.

a) E-mail: xuedanzhang@gmail.com

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3. Scheduling Algorithm

3.1 Explicit Conflicts

In CP-TDMA, time is divided into transmission slots of the same length, and grouped into sub-frames and frames. A frame consists of q sub-frames, and each sub-frame consists of l time slots.

In order to avoid explicit conflicts, we assign the links which have explicit conflicts to different sub-frames. Take the network in Fig. 1 for example, $L1$ and $L2$ are allocated to different sub-frames, so are $L3$ and $L4$. The sub-frame assignment for the links in Fig. 1 is shown in Fig. 2.

The assignment problem can be modeled as a standard *edge coloring* problem. In sub-frame scheduling, two links that have explicit conflicts are not assigned the same sub-frame. Similarly, in a valid edge coloring, no two edges incident on the same node are assigned the same color. The assignment for explicit conflicts can be obtained by mapping each sub-frame to a color and thus the minimum edge coloring is equivalent to the optimal scheduling for this type of conflicts.

Vizing's theorem [3] states that a valid edge coloring for a graph can be obtained by using at most $\Delta + \mu$ colors, where μ is the maximum multiplicity of an edge. However, the existing distributed edge coloring algorithms cannot achieve the optimal. Panconesi and Srinivasan [4] proposed a distributed edge coloring algorithm that uses at most $(2\Delta - 1)$ colors. They also proposed a better distributed algorithm [5] that requires at most $1.6\Delta + O(\log^{1+\Delta} N)$ colors. Grable and Panconesi [6] designed a randomized distributed edge coloring algorithm which uses at most $(1 + \varepsilon)\Delta$ colors, where $\varepsilon > 0$ is an input parameter. Their algorithm computes nearly optimal edge colorings very quickly but may fail to assign a valid color to each edge in the graph. Gandham et al. [7] developed a distributed implementation of Misra and Gries's centralized edge coloring method [8]. Their algorithm can obtain a valid edge coloring of acyclic graphs using at most $(\Delta + 1)$ colors. But their algorithm includes two complicated coloring rounds and needs large numbers of message exchanges by the nodes. The algorithms above are not suitable for distributed slot assignment in wireless ad hoc networks.

Algorithm **M** designed by Marathe et al. [9] inspired our proposed algorithm, which is a simple modification of the algorithm in [6] for edge-coloring simple graphs but with substantially improved performance. Unlike the original algorithm, this modified algorithm never fails. If the graph $G = (V, E)$ satisfies the condition $\Delta \gg \log N$, **M** can obtain the nearly optimal edge-coloring of G. Its implementation is rather simple and nodes work in parallel by exchanging information only with their immediate neighbors. Besides, its performance in terms of both running time and number of colors used is quite good.

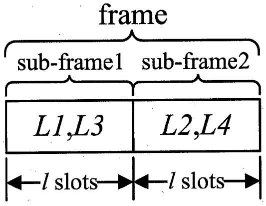


Fig. 2 An example of sub-frame assignment of CP-TDMA.

We develop an implementation of **M** for edge-coloring multigraphs at each wireless node and call it μ-**M**. The sender of each directed edge is in charge of coloring. Each edge has its own palette whose initial size is Δ, which is the number of colors allowed to be used. μ-**M** executes the following four steps repeatedly until all edges are colored.

1. Each sender chooses uniformly at random a tentative color from the corresponding palette for each of its uncolored links, and guarantees that any two links do not select the same color.
2. For each uncolored link, the sender sends a message describing the tentative colors to the receiver, and the receiver checks the tentative colors against the colors of its colored edges for possible color collision. If a collision occurs, the receiver sends a failure message back, with information on its occupied colors. If no collision occurs, the receiver sends a success message back and removes the color from the palettes of its uncolored links.
3. If a success message is received, the sender marks the link colored and removes the color from the palettes of its uncolored links. Otherwise, the sender removes the colors in the message (the receiver's occupied colors) from the palettes of the links.
4. If any palette is empty, one fresh new color is added to it.

In μ-**M**, an edge-coloring of G corresponds to a sub-frame assignment for explicit conflicts in a frame. Obviously, q is the upper bound of the number of colors used by μ-**M**. For the application of wireless ad hoc networks, if $\Delta \gg \log N$, q is set to $\Delta + \mu$. Otherwise, μ-**M** is expected to use between 5% and 20% more colors than $\Delta + \mu$ ($\mu = 2$), and hence q is set to 1.25Δ in CP-TDMA.

3.2 Implicit Conflicts

Implicit conflicts are a consequence of wireless interference between nodes within interference range. It is very difficult to describe this interference relation accurately because topology information out of transmission range is hard to capture precisely, and interference range irregularity [10] may change the interference relations among nodes over time. Moreover, the transmission assignments of the other nodes are unknown. Hence, it is a challenge to avoid implicit conflicts between interfering wireless links.

Our approach, which solves implicit conflicts, is to assign time slots to links probabilistically. After implementing μ-**M** to assign sub-frames, each link will pick one transmission time slot in the assigned sub-frame according to the designed policy.

Suppose there are M links in the network. For link i, there exist m_i $(0 \le m_i \le M)$ links which interfere with its transmission. Since the links know little information about each other, they pick a time slot independently. Whatever method is used to assign the time slot, the result can be modeled as follows. The probability that a link chooses the j-th time slot is p_j. We have $\sum_{j=1}^{l} p_j = 1$. Hence, the probability that link i transmits without collision is

$$p_s(i) = \sum_{j=1}^{l} p_j(1 - p_j)^{m_i} .$$

Theorem 1 (Cauchy's Inequality):

$$\left(\sum_{i=1}^{n} a_i b_i\right)^2 \le \left(\sum_{i=1}^{n} a_i^2\right)\left(\sum_{i=1}^{n} b_i^2\right),$$

where equality holds for $a_1/b_1 = a_2/b_2 = \cdots = a_n/b_n$.

According to Theorem 1, the probability of the i-th link's successful transmission $p_s(i)$ is maximized when $p_1/(1 - p_1)^{m_i} = p_2/(1 - p_2)^{m_i} = \cdots = p_l/(1 - p_l)^{m_i}$, i.e., $p_1 = p_2 = \cdots = p_l = 1/l$.

Consequently, our approach, which solves implicit conflicts, is for each link to pick one of l time slots in a sub-frame with equal probability. The approach is simple, but it is effective and optimal when the topology and neighbor information is unknown. In addition, the assignment can be done in a distributed manner.

The throughput G is defined as the number of successful packets in each time slot, i.e. the ratio of the number of successful packets in each frame to the frame length.

$$G = \frac{P_s}{ql}, \tag{1}$$

where P_s is the average number of all successful packets in each frame.

For the i-th link, define

$$x_i = \begin{cases} 1 & \text{if } i\text{-th link succeeds in a frame,} \\ 0 & \text{otherwise,} \end{cases}$$

then P_s can be written as,

$$P_s = E\left(\sum_{i=1}^{M} x_i\right) = \sum_{i=1}^{M} E(x_i)$$
$$= \sum_{i=1}^{M} \Pr(x_i = 1) = \sum_{i=1}^{M} p_s(i).$$

Hence, the throughput G is,

$$G = \frac{\sum_{i=1}^{M} p_s(i)}{ql} = \frac{\sum_{i=1}^{M} \left(1 - \frac{1}{l}\right)^{m_i}}{ql}.$$

Now l will be determined so that the throughput G is maximized. Suppose every link has the same number of underlying interfering links in the network, i.e. $m_1 = \cdots = m_M$, then $G = M(1 - 1/l)^{m_1}/ql$ and

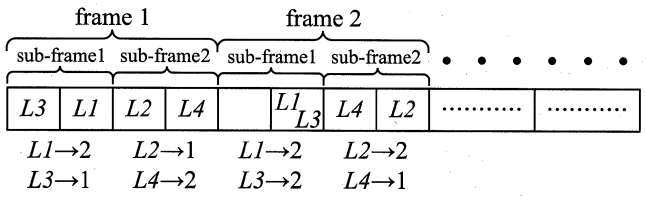


Fig. 3 CP-TDMA scheduling example.

$$\frac{\partial G}{\partial l} = \frac{M\left(1 - \frac{1}{l}\right)^{m_1}(m_1 + 1 - l)}{ql^2 - ql}.$$

Set $\frac{\partial G}{\partial l} = 0$, and we get the optimal value of l as

$l = m_1 + 1.$

If each link has a different number of underlying interfering links, l can be determined by the method of numerical analysis.

Figure 3 shows a CP-TDMA scheduling of links in Fig. 1. There is only one collision, namely, between $L1$ and $L3$ at the second time slot in sub-frame 1 of frame 2.

4. Performance Evaluation

The following scenarios are adopted for simulation:

Scenario 1: 40 nodes are placed uniformly within a 10 × 10 square area.

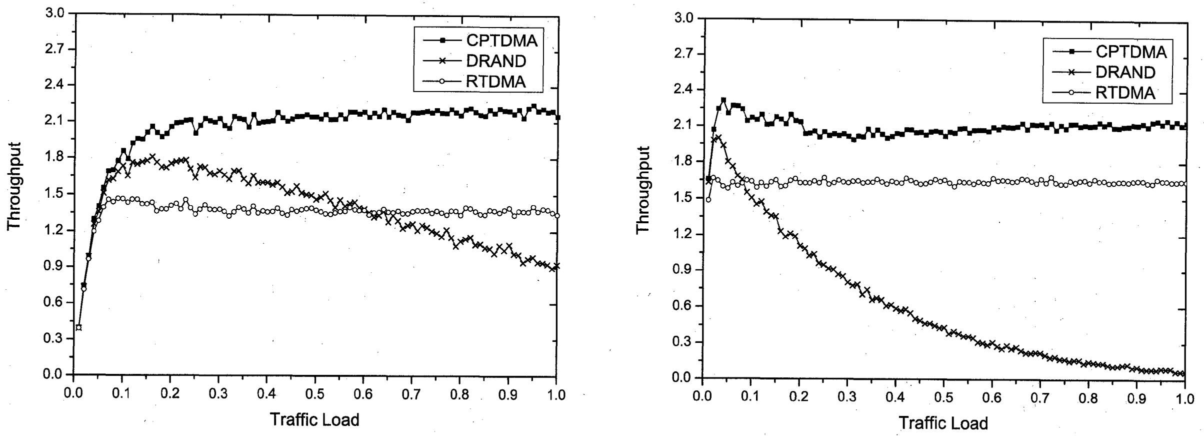
Scenario 2: 100 nodes are placed uniformly within a 10 × 10 square area.

The transmission range of the nodes is 1 and the interference range is 2.5 (the ratio of interference range and communication range is typically between 2 and 3 [11]). The network traffic load varies from 0 to 1. The simulations for all scenarios are implemented in C++ and the simulation results are averaged over 600 runs.

We compare the performance of CP-TDMA with the algorithms DRAND and R-TDMA in [12].

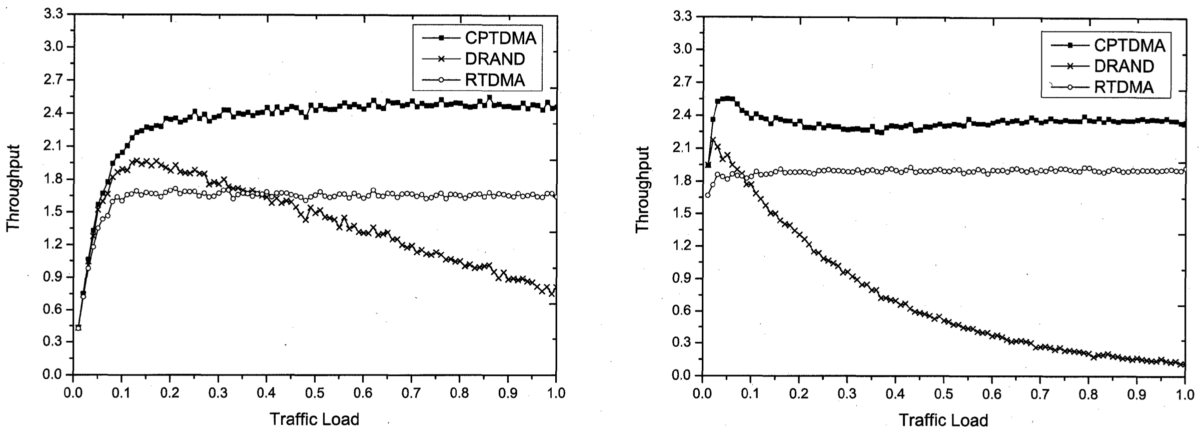
1. DRAND: Every node picks a timeslot which is not taken by its two-hop neighbors. DRAND requires only local (two-hop) information. It removes interferences only among those within a two-hop range. In fact, two nodes more than two hops from each other may be within the interference range. Besides, although two nodes may not communicate, they might still be in an interference range. DRAND does not capture these situations, so it exhibits poor performance when the traffic load is heavy.
2. R-TDMA: A node with a packet ready for transmission chooses a slot i between 0 and $F - 1$ with uniform probability, where F is the maximum number of links which interfere each other in the network.

Figure 4 shows the average throughput of CP-TDMA, DRAND and R-TDMA for Scenarios 1 and 2, respectively. From the simulation results, we can see that in both cases our algorithm performs significantly better than DRAND and R-TDMA. In addition, the performance of CP-TDMA



(a) Scenario 1. (b) Scenario 2.

Fig. 4 Throughput comparison with DRAND and R-TDMA under the protocol model.



(a) Scenario 1. (b) Scenario 2.

Fig. 5 Throughput comparison with DRAND and R-TDMA under the physical model.

is stable, i.e. as the density of the nodes and the traffic load increase, the throughput varies little. Note that the throughput of R-TDMA in Scenario 2 is higher than that in Scenario 1. This is because when the number of nodes increases, the distribution of links is more symmetrical and the spatial usage is improved.

Since the *physical* interference model [1] is widely used and we would like to observe how the effectiveness of our scheduling algorithm will change if the physical interference model is utilized, we also evaluate the algorithms by applying the physical model. The path loss exponent is set to be 2 and the SIR (Signal-to-Interference Ratio) threshold is set to be 8 dB.

Figure 5 shows the average throughput of CP-TDMA, DRAND and R-TDMA based on the physical model for Scenarios 1 and 2, respectively. It is obvious that CP-TDMA also exhibits good performance under the physical model. Note that the throughput based on the physical model is higher than that based on the protocol model. It is because the physical model is less restrictive than the protocol model: it may occur that a packet from node u to node v is correctly received even if there is a simultaneous transmitting node w within the interference range of v (for instance, because node u is very close to node v). As a result, higher throughput is achieved by applying the physical model.

5. Conclusion

In this paper, we propose a new distributed coloring- and probability-based TDMA scheduling that maximizes the throughput of wireless ad hoc networks. We evaluated our scheduling algorithm analytically and numerically and showed that it exhibits better performance than existing algorithms.

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