Topology-Transparent Distributed Multicast and Broadcast Scheduling in Mobile Ad Hoc Networks

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Abstract—Transmission scheduling is a key problem in mobile ad hoc networks. Many transmission scheduling algorithms have been proposed to maximize the spatial reuse and minimize the time-division multiple-access (TDMA) frame length in mobile ad hoc networks. Most algorithms are dependent on the exact network topology and cannot adapt to the dynamic topology in a mobile wireless network. To overcome this limitation, several topology-transparent scheduling algorithms have been proposed. The slots are assigned to guarantee that there is at least one collision-free time slot in each frame. In this paper, we consider multicast and broadcast, and propose a novel topology-transparent distributed scheduling algorithm. Instead of guaranteeing at least one collision-free transmission, the proposed algorithm guarantees one successful transmission exceeding a given probability, and achieves a much better average throughput. The simulation results show that the performance of our proposed algorithm is much better than the conventional TDMA and other existing algorithms in most cases.

I. INTRODUCTION

Scheduling medium access in mobile wireless ad hoc networks is challenging because of node mobility, and the limited availability and variability of wireless bandwidth. In the conventional time-division multiple-access (TDMA) networks, each node is assigned a unique time slot to transmit. This works well when the connectivity information among the nodes is known and the number of nodes in the network is not large [4]. While in mobile ad hoc networks, the number of nodes is much larger than the number of neighbours of a node, and the system performance can be greatly improved by applying spatial reuse. Previous approaches in topology-dependent scheduling require each node to maintain accurate network topology information. This is impractical in wireless ad hoc networks and thus such approaches are not adaptive to dynamic topology changes. On the other hand, existing topology-transparent scheduling methods provide a guaranteed minimum throughput bound with relatively low network utilization. In addition, most of them are only applicable to unicast transmission.

We focus on the TDMA networks, in which time is divided into equal-sized transmission slots, which are grouped into frames. Each slot is designed to accommodate the transmission of one equal-sized packet and a guard time.

In this paper, we propose a novel topology-transparent distributed scheduling algorithm in the TDMA networks with a different design strategy. We study multicast and broadcast. The main contributions of our work are as follows. First, different from most existing work that guarantees at least one successful conflict-free time slot, our proposed scheduling algorithm guarantees that a multicast or broadcast transmission succeeds within a frame time with a high probability, thus achieving a much higher average throughput than existing algorithms. Second, we run extensive simulations to illustrate that our proposed algorithm performs much better than other existing algorithms for most cases.

The remainder of this paper is organized as follows. The related work is presented in Section II, and we introduce our system model and definitions in Section III. We describe our proposed algorithm in detail and analyze it in Section IV. In Section V, we compare our proposed algorithm with other existing algorithms both analytically and by simulations. We summarize and conclude the paper in Section VI.

II. RELATED WORK

The related work on transmission scheduling can be categorized into two different groups: topology-dependent and topology-transparent, based on whether the scheduling algorithm depends on the detailed network topology. Existing topology-dependent approaches focus on finding a conflict-free schedule based on the detailed network topology. Thus, re-computation and information exchanges are required to maintain the accurate network topology information and distribute the new schedules when the network topology changes. Thus, the robustness and effectiveness of these topology-dependent scheduling algorithms are undermined in large, highly dynamic wireless mobile ad hoc networks.

To overcome the aforementioned disadvantages of topology-dependent scheduling approaches, topology-transparent scheduling algorithms have been proposed. Chlamtac and Farago [3] developed a topology-transparent algorithm that guarantees at least one collision-free time slot in each frame, but the performance is even worse than the conventional TDMA in some cases. Ju and Li [10] proposed another algorithm to maximize the minimum guaranteed throughput. However, it only considers unicast communication. Cai et al. [2] proposed a broadcast scheduling algorithm, modified Galois field design (MGD), which sends

the same message during one frame to guarantee exactly one successful broadcast transmission per frame. The throughput is relatively small, since the maximum number of transmission is one in a frame. Sun et al. [12, 13] designed an acknowledgement-based scheduling protocol for multicast and broadcast, which improves the expected throughput. Unfortunately, the overhead introduced by acknowledgements increases linearly with the frame length, degrading the performance dramatically especially when the total number of nodes and the maximum number of neighbours of a node are large. Farnoud and Valaee [8] applied positive orthogonal codes to design a reliable broadcast algorithm for safety message in vehicular networks, but it focused on a specific application and network topology (one-dimensional roads). The algorithm requires that each node is location-aware and the performance metric is the successful probability when the traffic load is not heavy.

III. SYSTEM MODEL AND DEFINITIONS

A mobile ad hoc network can be represented by a graph $G(V, E)$. V is the set of all network nodes and E is the set of all edges indicating which pairs of nodes interfere with each other. The degree of a node v, $D(v)$, is defined as the number of nodes in v's interference range. The maximum degree D_{max} is much smaller than the number of nodes N. D_{max} is assumed to remain constant while the network topology changes [9].

When nodes communicate, they may suffer two types of conflicts [7]. The first one, called primary conflict, refers to the situation that a transmitting node cannot receive a packet at the same time slot. The second one, called secondary conflict, refers to the situation that a node cannot receive more than one packet in a time slot. We assume that a reception failure is only due to transmission collision. The transmission from Node w to Node v succeeds when 1) Node v is not transmitting, and 2) other nodes in v's interference range are not transmitting.

In the TDMA networks, an acknowledgement mechanism can be easily implemented when we consider the unicast traffic only. However, it is not true in multicast and broadcast communications. Note that, for multicast and broadcast, the acknowledgements from different intended receivers may suffer the secondary conflict and collide at the transmitter, leading to the failure of the acknowledgement mechanism. In this paper, we do not use the acknowledgement mechanism and only transmit one multicast or broadcast packet during one frame.

As in [3], [10], using the number of nodes in the network N and the maximum node degree D_{max} as two design parameters, we propose a distributed, topology-transparent scheduling algorithm, but we focus on multicast and broadcast.

In multicast, a node transmits a message to some of its neighbours. We denote these neighbours by *intended receivers*. In broadcast, all the neighbours of a node are the *intended receivers*.

Definition 1 Successful multicast (broadcast) transmission: A multicast (broadcast) transmission is successful if and
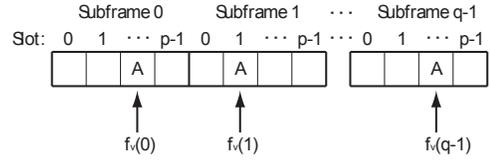

Fig. 1.   The Frame Structure.

only if all of its intended receivers (neighbours) receive the transmitted message successfully.

Definition 2 Average throughput: The average throughput is defined as the probability that a multicast or broadcast transmission succeeds within one frame divided by the frame length.

IV. PROPOSED DISTRIBUTED, TOPOLOGY-TRANSPARENT ALGORITHM

Consider a polynomial over $GF(p)$, $\sum_{i=0}^{k} a_i x^i (\mod p)$, where p is a prime or a prime power and $a_i \in \{0, 1, 2, \ldots, p-1\}$. The equation $\sum_{i=0}^{k} a_i x^i (\mod p) = 0$ has at most k integral roots over $\{0, 1, 2, \ldots, p-1\}$ [6].

Consider a single-channel TDMA network $G(V, E)$ with N mobile nodes and the maximum node degree D_{max}. We divide a TDMA frame into q subframes, each of which consists of p fixed-length time slots, i.e., the frame length is pq time slots. Each node v is assigned a unique polynomial with degree k mod p, $f_v(x) = \sum_{i=0}^{k} a_i x^i (\mod p)$, where $v \in V$, as its time slot assignment function (TSAF). Let a standard row vector S be $\{0, 1, 2, \ldots, q-1\}$. $f_v(S) = \{f_v(0), f_v(1), f_v(2), \ldots, f_v(q-1)\}$ is known as the time slot location vector (TSLV) for Node v. Thus, Node v transmits in the time slot $f_v(i)$ in Subframe i, where $i \in \{0, 1, 2, \ldots, q-1\}$. Each node therefore has q transmissions in one frame [10]. The frame structure is shown in Fig. 1.

The number of TSAFs should be equal to or greater than the number of nodes in the network. Thus, $p^{k+1} \geq N$ is required. Based on coding theory [11], if $q \leq p$ is satisfied, any two nodes have at most k conflicts during one frame. Interested readers can refer to [3] and [10] for a detailed proof. Since each node can have at most D_{max} neighbours, the maximum number of possible collisions of one node in a frame is upper-bounded by kD_{max}. Most of the algorithms [2, 3, 10, 12, 13] satisfy the inequality $q \geq kD_{max} + 1$ to guarantee that every node can transmit data to any neighbour in at least one slot during one frame. Actually, the upper bound kD_{max} is loose in most cases due to the following reasons: 1) the number of neighbours of a node may be less than D_{max}, 2) any two nodes have no conflicts during one frame if the difference of their TSAFs is a constant [10], and 3) the common roots of two TSAFs, which are equal to or greater than q, do not result in conflicts between two nodes, since $q \leq p$. Thus, the inequality $q \geq kD_{max} + 1$ results in poor average throughput especially for multicast and broadcast. We

are interested in finding the optimal q, where $q \leq kD_{max}$, to maximize the average throughput as well as guaranteeing one successful multicast or broadcast transmission during one frame exceeding a given probability.

We first make a basic assumption and introduce our algorithm for multicast and broadcast, respectively.

Assumption: For an arbitrary node v, the conflicts caused by its neighbours are assumed to be independent.

Note that, given k and p, each node selects a unique TSAF from a set of p^{k+1} polynomials. Correlation thus exists among the conflicts caused by different nodes. We assume that this correlation can be neglected, under which we analyze our proposed algorithm. The simulation results shown in Section V demonstrate that this assumption does not affect the accuracy of our results.

A. Multicast

A node v multicasts a message to its R intended receivers w_i, where $i = 1, 2, \ldots, R$. Let A_i^j be the indicator that Node w_i receive node v's message successfully in Subframe j.

$$
\begin{aligned}
P(A_i^j = 1) &= 1 - P(A_i^j = 0) \\
&= 1 - \frac{1}{p} - (1 - \frac{1}{p})[1 - (1 - \frac{1}{p})^{D(w_i)-1}](1) \\
&\geq (1 - \frac{1}{p})^{D_{\max}},
\end{aligned}
$$

where the probability that a node transmits in a time slot is $\frac{1}{p}$, since a TSAF over $GP(p)$ is uniformly distributed over $\{0, 1, 2, \ldots, p-1\}$ [5].

Let B_i be the indicator that Node w_i receives a multicast message from Node v in one frame, where a frame has q subframes. The probability that the event B_i happens can be expressed as,

$$
\begin{aligned}
P(B_i = 1) &= 1 - P(A_i^0 = 0, A_i^1 = 0, \ldots, A_i^{q-1} = 0) \\
&= 1 - [1 - P(A_i^j = 1)]^q \quad (2) \\
&\geq 1 - [1 - (1 - \frac{1}{p})^{D_{max}}]^q.
\end{aligned}
$$

Note that a multicast message is successful if and only if all its R intended receivers receive the message in one frame. Thus, we can obtain the probability that a multicast message is received successfully as follows:

$$
\begin{aligned}
P(Success) &= P(B_1 = 1, B_2 = 1, \ldots, B_R = 1) \\
&= P^R(B_i = 1) \quad (3) \\
&\geq [1 - (1 - (1 - \frac{1}{p})^{D_{\max}})^q]^R.
\end{aligned}
$$

Denote the average throughput and the minimum average throughput by T_m and G_m, respectively. We can get:

$$
T_m = \frac{P(Success)}{pq} \geq G_m = \frac{[1 - (1 - (1 - \frac{1}{p})^{D_{\max}})^q]^R}{pq}. \quad (4)
$$

Thus, the goal of our proposed algorithm is to find the optimal value of q to maximize the value of G_m, while guaranteeing that the probability of successful multicast transmission during one frame time is at least Φ.

Theorem 1: For a given k, the optimal value of q that maximizes the value of G_m and satisfies $P(Success) \geq \Phi$, q_{opt}, is:

$$
q_{opt} = \begin{cases} \arg \max\limits_{q \in \{\lfloor q_1 \rfloor, \lceil q_1 \rceil\}} G_m(q), & \text{if } q_2 \leq \lfloor q_1 \rfloor \\ \lceil q_2 \rceil, & \text{otherwise} \end{cases}, \quad (5)
$$

in which:

$$
\begin{aligned}
q_1 &= \frac{\ln x_0}{\ln[1 - (1 - \frac{1}{p})^{D_{\max}}]} \\
q_2 &= \frac{\ln(1 - \Phi^{\frac{1}{R}})}{\ln[1 - (1 - \frac{1}{p})^{D_{\max}}]},
\end{aligned}
$$

where x_0 is the unique root in $(0, 1)$ of the equation $Rx \ln x + 1 - x = 0$.

Proof: Since the probability that a multicast transmission is successful is no less than Φ, i.e.,

$$
[1 - [1 - (1 - \frac{1}{p})^{D_{\max}}]^q]^R \geq \Phi, \quad (6)
$$

we have:

$$
q \geq q_2 = \frac{\ln(1 - \Phi^{\frac{1}{R}})}{\ln[1 - (1 - \frac{1}{p})^{D_{\max}}]}. \quad (7)
$$

From (4), for a given k, we solve the following equation to find the optimal value of q that maximizes the value of G_m:

$$
\frac{\partial G_m}{\partial q} = -\frac{(1 - x)^{R-1}}{pq^2}(Rx \ln x + 1 - x) = 0, \quad (8)
$$

where $x = [1 - (1 - \frac{1}{p})^{D_{\max}}]^q$ and $0 < x < 1$. We set $g(x) = Rx \ln x + 1 - x$ and obtain $g'(x)$ as:

$$
g'(x) = R \ln x + R - 1. \quad (9)
$$

Thus, $g(x)$ increases with x when $x > e^{\frac{1-R}{R}}$ and decreases with x when $x < e^{\frac{1-R}{R}}$. Note that $g(0) = 1$ and $g(1) = 0$. There must be a unique root x_0 located in $(0, 1)$. Therefore, the maximum value of G_m is achieved when $q = q_1 = \frac{\ln x_0}{\ln[1-(1-\frac{1}{p})^{D_{\max}}]}$. G_m increases with q when $q < q_1$ and decreases with q when $q \geq q_1$. Note that the number of subframes in one frame time, q, is an integer. Hence, if $q_2 \leq \lfloor q_1 \rfloor$, the maximum value of G_m is achieved when $q = \lfloor q_1 \rfloor$ or $q = \lceil q_1 \rceil$. Otherwise, the maximum value of G_m is achieved when $q = \lceil q_2 \rceil$. ∎

Note that q_2 is less than or equal to p. From (7), we obtain that $[(1 - (1 - \frac{1}{p})^{D_{\max}})]^p \leq 1 - \Phi^{\frac{1}{R}}$, i.e., $p \geq p^*$, where p^* is the root of $[(1 - (1 - \frac{1}{p})^{D_{\max}})]^p = 1 - \Phi^{\frac{1}{R}}$. Recall that p decreases with k. Thus, for the given N and D_{max}, we can obtain that $k \leq k^*$, where k^* is the largest integer satisfying $p \geq p^*$.

Thus, we design an optimal topology-transparent scheduling algorithm that maximizes the average throughput and guarantees that a multicast transmission is successful during one frame with a high probability as follows.

1) For the given N and D_{max}, use Theorem 1 to choose k and p, where p is the smallest prime or prime power that satisfies $p^{k+1} \geq N$, and then select the optimal q.

2) Each node is randomly assigned with a unique degree k TSAF.

3) Each node calculates its TSLV according to the method introduced in Section III.

4) Each node transmits its data packets only at its assigned slots.

B. Broadcast

Broadcast is a special case of multicast. All the neighbours of Node v are the intended receivers, i.e., $R = D(v)$. Substituting R in Theorem 1 with D_{max}, we can obtain similar results for broadcast.

V. PERFORMANCE EVALUATION

In this section, we use the average throughput as the performance metric and quantitatively compare our proposed scheduling algorithm, both analytically and by simulation, with the conventional TDMA fixed assignment scheme, the MGD algorithm proposed by Cai et al. [2] and the acknowledgement-based algorithm proposed by Sun et al. [12, 13], referred as Sun's algorithm. We study the impact of different configurations of R, N, D_{max}, and Φ on the performance of our algorithm. Since broadcast is a special case of multicast, we investigate the performance for multicast and broadcast simultaneously.

A. Simulation Setup

We adopt the Gauss-Markov mobility model [1], which has been shown to be more realistic than the widely used Random Waypoint model. All nodes are initially distributed uniformly and at random in a region of 1000 m × 1000 m. The tuning parameter α, which is used to present different levels of randomness, is set to 0.5 (Brownian motion is obtained by setting α to zero and the linear motion is obtained by setting α to one). The speed follows a Gaussian distribution. The mean and standard deviation of the speed are set to 0.9 ms^{-1} and 0.5 ms^{-1} [15].

We apply the optimal k, p, and q that achieve the maximum value of average throughput derived in Section IV. We run each simulation for 500 times.

Unlike our proposed algorithm and the other two algorithms we refer to, Sun's algorithm is acknowledgement-based. To make a fair comparison, we must consider the overhead introduced by employing acknowledgements in Sun's algorithm. We adopt the same parameters used in [12]. In Sun's algorithm, a single time slot is divided into two segments, namely, the data segment and the acknowledgement segment. The acknowledgement segment is further divided into pq mini-slots for acknowledgement transmission. Each of these mini-slots is used to accommodate the transmission of an acknowledgement of four bytes. Under the IEEE 802.11 standard, the data size is 2272 bytes [14]. Thus, the overhead introduced by employing acknowledgements is $\frac{4pq}{2272}$.

B. Analytical and Simulation Results

First, we investigate the effect of R on the performance of our proposed algorithm. Then, for a fair comparison with other algorithms designed for broadcast, we set the number of intended receivers to the maximum node degree, i.e., $R = D_{max}$, so as to evaluate the effect of N, D_{max}, and Φ.

1. Effect of R on Performance

Given that $N = 1024$, $D_{max} = 14$, and $\Phi = 0.99$, we vary the number of intended receivers R from two to 14, to study the performance of our proposed algorithm. Since the overhead introduced by employing the acknowledgement mechanism in Sun's algorithm only depends on N and D_{max}, rather than the number of intended receivers R, we do not include Sun's algorithm here. In Fig. 2, we observe that our proposed algorithm achieves much better average throughput than MGD and the conventional TDMA. Even when $R = D_{max}$, the average throughput of our algorithm is almost twice and four times greater than that of MGD and the conventional TDMA, respectively. The simulation results match our analytical results very well. With increasing R, a larger q is necessary to guarantee that a multicast transmission succeeds with probability no less than Φ, thereby reducing the average throughput.

2. Effect of N on Performance

Given that $D_{max} = 14$, $R = D_{max} = 14$, and $\Phi = 0.99$, and N is configured with nine different settings from 200 to 1000, we evaluate the average throughput of our proposed algorithm, and compare it with Sun's algorithm, MGD, and the conventional TDMA. In Fig. 3, both the analytical and simulation results show that our proposed algorithm performs much better than the other algorithms. Sun's algorithm performs relatively better than MGD and the conventional TDMA when N is small, but its performance degrades dramatically when N becomes large. When N is greater than 600, the performance of Sun's algorithm becomes worse than MGD. This is because the overhead introduced by acknowledgements is high when N is large. Besides, we can observe that the performance of our proposed algorithm deteriorates slowly with increasing N, implying that the performance of our algorithm is not sensitive to the number of nodes in the network.

3. Effect of D_{max} on Performance

Given that $N = 1024$, $\Phi = 0.99$, and $R = D_{max}$, we investigate the performance of our algorithm with 11 D_{max} settings from 6 to 26. A larger D_{max} indicates that the network is denser and there are more possible conflicts. As shown in Fig. 4, we can see that our algorithm outperforms the other three algorithms. The performance of Sun's algorithms is quite good when D_{max} is small, but degrades dramatically when D_{max} becomes large. Compared with Fig. 3, we can observe that the maximum node degree D_{max} has a greater impact on the performance than N, the number of network nodes. Again, the simulation results closely match our analytical results.

4. Effect of Φ on Performance

Given $N = 1024$ and $R = D_{max} = 14$, we evaluate the effect of Φ on the performance of our proposed algorithm. We investigate the average throughput of our proposed algorithm
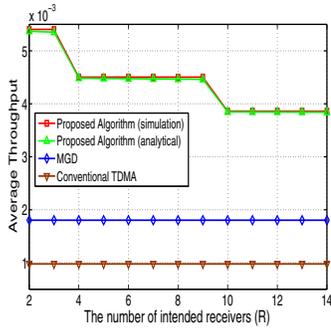
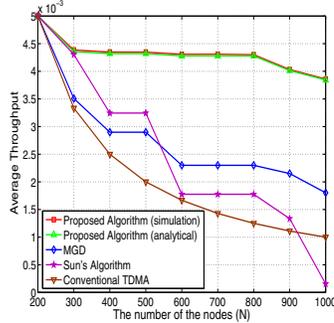Fig. 2. The effect of R on the performance.
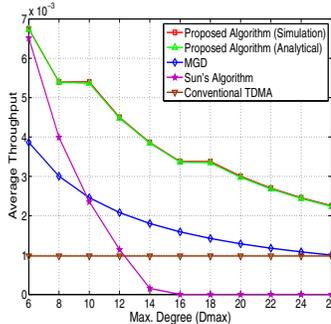

Fig. 3. The effect of N on the performance.

Fig. 4. The effect of $D_{\max}$ on the performance.

under four different Φ, namely, 0.90, 0.95, 0.97, and 0.99. Recall that we need to guarantee that a multicast or broadcast transmission succeeds during one frame with probability no less than Φ. Thus, a longer frame may be necessary to guarantee a successful multicast or broadcast transmission with a higher probability Φ. In Fig. 5, both the analytical and simulation results show that the average throughput drops with increasing Φ, which concurs with our analysis.

VI. CONCLUSION

In this paper, we propose a topology-transparent distributed scheduling algorithm for multicast and broadcast in wireless mobile ad hoc networks. Unlike most of the existing work, our proposed algorithm does not guarantee at least one successful transmission during one frame. Instead, our algorithm guarantees that a multicast or broadcast transmission succeeds during one frame with high probability, thus achieving much
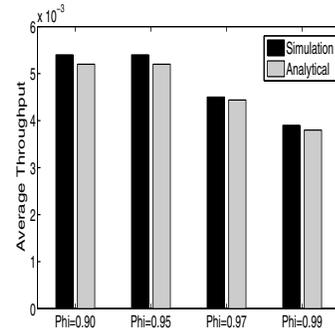

Fig. 5. The effect of Φ on the performance.

better average throughput. We show that our proposed algorithm outperforms the conventional TDMA and other existing algorithms both analytically and by extensive simulations.

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